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An Argument For A Neutral Free Logic

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AN ARGUMENT FOR A NEUTRAL FREE LOGIC

by

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of Wayne State University,

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DEDICATION

For James and Kathryn

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TABLE OF CONTENTS

DEDICATION	ii
ACKNOWLEDGMENTS	iii
Introduction: Master Argument, Chapter Summaries, and Closure	1
Motivation and Explanation of NFL	1
Chapter Summaries	5
Closure of the A Priori	29
Chapter One: The Existential Quantifier and Ontological Commitment	39
Paraphrase and Ontology	39
Avoiding Ontological Commitment	41
Sentences Meeting I-III are Best Represented by an Objectual Quantifier	45
Sentences Meeting I-III are Univocal	53
Sentences Meeting I-III are Heavyweight	57
Objection One: The Triviality Argument	59
Objection Two: From the Relative Strengths of Realist Claims	61
Objection Three: Ambiguous Domain	66
Two kinds of Sentence that Satisfy I-III	68
Chapter Two: Positive Free Logic	72
Bivalent PFL and the Prototype Schema	72
An Appeal to Authority	74
Ambiguous Quantifiers Again?	78
Non-bivalent PFL: Supervaluations	87
Introduction to SVS	87
The Utility Defense of SVS	92

Criticisms of SVS	98
Accounting for CLTs with Corresponding Arguments	112
Saving Contingent Truths	122
Chapter Three: Negative Free Logic	128
Introduction to Negative Free Logic	129
Three Criticisms of Negative Free Logic	131
Three Arguments for Negative Free Logic	134
Bivalence	138
Williamson's Argument for Bivalence	139
Another Defense of Bivalence: Argument from the T—Scheme	146
The Law of Excluded Middle	149
Defenses of LEM	150
Criticisms of LEM	152
Chapter Four: Neutral Free Logic	158
Choices for a Neutral Free Logic	158
Domain	158
Identity	162
Relations and Symbols	164
Quantifiers	166
Logical Properties	170
The Semantic Rules	173
Criticism: Hedging	177
General problems	177

Pryor's account	180
Use and Mention	185
BIBLIOGRAPHY	188
ABSTRACT	197
AUTOBIOGRAPHICAL STATEMENT	199

INTRODUCTION: MASTER ARGUMENT, CHAPTER SUMMARIES AND CLOSURE

I will argue that the appropriate semantics for natural language is a neutral free semantics with weak Kleene tables and trivalent quantifiers. The general pattern of my project will be to present an argument with an absurd conclusion and consider the options for avoiding the absurdity. I will try to show that all but one of these options are unacceptable. In the introduction I will present the argument, lay out the possible ways of criticizing it, and explain the approach I endorse. After that, I will summarize the arguments of the following chapters and provide an argument for a premise of the master argument that does not fit neatly elsewhere.

Motivation and Explanation of Neutral Free Logic—The Master Argument

Call a sentence of the form ‘ c exists’ a *simple existence claim*. If an appropriate symbolization of a simple existence claim is ‘ $\exists x(x = c)$ ’ then it is a problem for classical logic that it regards all simple existence claims as logically true. Logical truths are knowable a priori and simple existence claims are not. Michael McKinsey (in conversation) and James Pryor¹ have suggested an argument similar to what I will call the ‘master argument.’

1. It is logically true that Powers is Powers
2. Logical truths are a priori knowable.
3. So, it is a priori knowable that Powers is Powers.
4. That Powers is Powers logically entails that Powers exists.
5. What is logically entailed by a priori knowable truths is a priori knowable.²

¹ James Pryor, "XIII--Hyper-Reliability and Apriority," *Meeting of the Aristotelian Society*, (2006): 335-6. Pryor does not formalize his argument, but a rough paraphrase of it would be: ‘Jack is self-identical’ implies that ‘Jack exists.’ Since the latter seems not to be knowable a priori, the former is not either. Since logical truths are knowable a priori, ‘Jack is self-identical’ must not be one.

² This is just a weakened version of Michael McKinsey’s principle: *Closure of Apriority under Logical Implication* (CA) from his “Forms of externalism and privileged access,” *Philosophical Perspectives* 16, (2002): 207. His is prefixed with ‘necessarily’ and uses a ‘knows a priori’ relation between people and propositions instead of my ‘a

6. So, it is a priori knowable that Powers exists.

The conclusion is absurd even supposing that ‘Powers’ refers to Larry Powers, the erstwhile professor of philosophy at Wayne State University and author of *Non-contradiction*. The alternatives are: to accept the absurd conclusion, to reject the validity of the argument from 1 and 2 to 3 or of the argument from 3, 4, and 5 to 6, or to reject a premise (either 1, 2, 4, or 5). The first two of these options are disastrous. Since ‘Powers’ in the argument above could be replaced with *any* name, accepting the conclusion is accepting that *all* simple existence claims are knowable a priori. With a few faintly possible exceptions, it is plausible that *no* simple existence claims are knowable a priori. The second option is equally undesirable; the inferences are intuitively valid. So much so that any view that rejects them is dubious, for that reason alone. Both inferences are of the form All A are B; x is an A; so, x is a B. For a defense of the validity of this argument form see Chapter Two, on positive free logic. So the remaining options are to reject 1, 2, 4, or 5. First consider premise 2.

That logical truths are knowable a priori has its own motivation: logic is the study of correct inferences, and determinations of correct inferences are purely mental activities; so, logic is a priori. However, 2 follows from 5; so it needs no defense above that of 5. Since all logical truths follow from all truths, including a priori-knowable truths, the closure principle 5 implies that all logical truths are knowable a priori. At the conclusion of this introduction, I will briefly offer a pair of arguments for 5 and defend them from criticism. If those arguments are successful then the only remaining options are to reject 1 or reject 4.

My defense of premise 4 comes from two premises and the transitivity of logical entailment:

priori knowable’ operator. The reductio argument above is very similar to the reductio argument for which McKinsey uses (CA) in that paper and elsewhere.

- (EG) That Powers is Powers logically entails that something is Powers.
- (OC) That something is Powers logically entails that Powers exists.

A rejection of premise 4 is a rejection of Existential Generalization or of the ontological commitment of simple existentially quantified claims. In chapter one I will defend (OC) by arguing that certain simple existentially quantified claims are ontologically committed. In chapter two I will consider two ways of denying the validity of (EG): positive free logic and supervaluational logic. Both allow that some statements containing non-referring individual constants can be true. I will argue that both of those views are inadequate and I will suggest that any acceptable semantics will disallow the truth of statements with empty constants. Since not all instances of the schema ' $x = x$ ' are true, ' $a = a$ ' is not a logical truth. So 1 must be rejected.

Premise 1 follows from ' $\forall x(x = x)$ ' by an application of Universal Instantiation. So a rejection of 1 will also be a rejection of the universally quantified claim or of UI. There are two ways to reject ' $a = a$ '. We could say that simple statements with empty names are false or that they are neither true nor false. A logic that regards those statements as false is a negative free logic (NgFL); one that makes ' $a = a$ ' neither true nor false (when 'a' is empty) is a neutral free logic (NFL). Notably, on a negative free logic ' $Pa \vee \sim Pa$ ' remains logically true but a neutral free logic loses that and all other classical tautologies containing individual constants. My particular NFL, because of its treatment of the empty domain, loses **all** logical truths, but that result is not required of NFL in general.

If there are persuasive arguments for bivalence then a negative free logic would be preferable to one allowing a lack of truth value. In chapter three I will consider arguments for

bivalence; most notably, I will consider Timothy Williamson's.³ I will also try to justify the loss of the Law of the Excluded Middle (LEM) by NFL. Doing so, in addition to finding fault with arguments for Bivalence will undermine support for negative free logic, and leave only the question of exactly what flavor of neutral free logic to accept.

Once one accepts a neutral free logic there are several substantive choices to make, including whether to use weak or strong Kleene truth tables, and whether to use bivalent or trivalent quantifiers. Truth functional connectives are *weak* Kleene connectives when any input that is neither true nor false results in an output that is neither true nor false. Strong Kleene tables are the same as weak for negation. However, for conjunction the value of the whole sentence is the minimum value of the inputs where $\text{false} < \text{neither} < \text{true}$. The value of a disjunction is the maximum value of its disjuncts. To get a bivalent existential quantifier, its rule must make it true under the circumstances that would make it true ordinarily (some assignment of domain elements to the formula's free variables is true) and false otherwise. For a trivalent existential quantifier, if every assignment of domain elements to free variables in a formula is false then the existentially quantified sentence is false. If some assignment results in a sentence that is neither true nor false then the quantified sentence is neither. Otherwise the sentence is true.

There is an oft-cited problem with accepting either version of the Kleene tables; neither logic has any sentential tautologies⁴. This 'problem' is really the benefit that motivates us to accept such a logic, since any sentence containing a name (purported tautology or not) lacks truth value. By an argument similar to the one above (1-6) the object designated by that name would,

³ The argument I consider appears in *Vagueness* (New York, Routledge, 1994): 188.

⁴ For example, Graham Priest mentions it in *An Introduction to Non-Classical Logic* (New York, Cambridge, 2001): 121.

absurdly, be known to exist a priori. Alternatively, one could, as Lehmann suggests,⁵ weaken the notion of logical truth to *not false on any interpretation* to recover the logical truths. However, it is clearly an error to accept that some logical truths might not be true.

The existence of objects ought not be possible to establish by logic alone, so a logical system ought not rule out the possibility of an empty domain. Allowing the empty domain causes problems and I will have to argue that the problems caused by my approach are not as bad as the problems avoided by it.

I plan to use chapter four to present in technical detail the exact form of the neutral free semantics I advocate and defend it from the other NFL competitors.

Chapter Summaries

Before briefly defending the epistemic closure principle, I present a more detailed summary of the following chapters.

Chapter One Summary—The Existential Quantifier and Ontological Commitment

One way of responding to the argument above is to object to premise four on the grounds that ‘something is identical to Powers’ does not entail that Powers exists—that is, to deny the ontological commitment of the existential quantifier (where a sentence P is ontologically committed to c when P necessitates ‘c exists’). In chapter one I argue of a circumscribed class of sentences including ‘something is identical to Powers’ that it bears ontological commitments. This argument is an adaptation of an argument from van Inwagen.

If a sentence is a simple existentially quantified claim (e.g., ‘there are Fs’, or ‘something is identical to c’) without a more fundamental paraphrase, and is equivalent to a numerically quantified claim then it bears ontological commitments. I argue for this primarily by ruling out

⁵ Scott Lehmann, “Strict Fregean Free Logic,” *Journal of Philosophical Logic* 23 (1994): 310. Lehmann also makes the suggestion in “More free logic.” in *Handbook of Philosophical Logic*, second edition, Volume 5, eds. D. Gabbay and F. Guenther (Dordrecht: Kluwer, 2002): 234.

alternatives. The first plausible alternative is that the quantifier in the fundamental, simple, and existential claim is best understood as a substitutional quantifier rather than as an objectual quantifier. This objection would undermine the ontological commitment of the sentence because the semantic rule for a substitutional, existential quantifier makes a sentence prefixed by it true in case there is a **linguistic entity** that could be substituted into the open sentence to make it true; this contrasts with the objectual interpretation in which existentially quantified sentences are true when some **domain element** satisfies the open sentence prefixed by the binding quantifier. So the objection goes, ‘something is identical to Powers’ is true because the name ‘Powers’ when inserted into ‘Something is identical to...’ results in a truth. There need not be any referent of the name upon which the truth of the existential claim rests. Consequently, ‘Powers exists’ need not follow.

It has long been known that undercounting is a problem for substitutional quantifiers.⁶ Suppose that the substitution class is names and that all neutrinos are unnamed. In such a circumstance ‘there is a neutrino’ would not be made true by any substitution of a name into ‘...is a neutrino.’ So, the substitutional quantifier rule would make the sentence false. This problem might be reparable by expanding the substitution class to include not only names but also descriptions; arguably, there is always one available so the undercounting problem is avoided. Unfortunately, this modification solves one problem only by creating another. There is now an overcounting problem. Since many descriptions are apt to denote, say, the president of the United States, the substitutional rule will make it false that ‘there is at most one president of the United States.’ This follows because, in the absence of a workable, objectual, semantic rule for identity claims, ‘x is POTUS and y is POTUS and $x \neq y$ ’ is made true by different descriptions substituted for x and y. The most likely candidate for a non-objectual rule for

⁶ For example, see W. V. O. Quine “Ontological Relativity,” *Journal of Philosophy* 65, no. 7 (1968); 210.

identity of descriptions is inter-substitutability, but this criterion for identity falters on examples like ‘he won the election because he was *the candidate with the craftiest staff*’; substitute ‘the winner of the election’ for ‘the candidate with the craftiest staff’.

The second plausible alternative to ontological commitment is an ambiguous objectual quantifier. On one reading the objectual existential quantifier would be committing—call that a *heavyweight* quantifier—and on another reading—the *lightweight* reading—it would not be committing. Appealing though this might be, it is unsuccessful for simple existential claims that are equivalent to numerical claims. Such claims are unambiguous. There is no reading of ‘at least one thing is identical to c’ that is not equivalent to ‘the number of things identical to c is one.’ If the latter sentence were heavy/light ambiguous then there would be non-contradictory readings of sentences like the ‘the number of Fs is exactly one and the number of Fs is not one.’ Also, it would be unacceptable to add or subtract some numerical claims. For example, ‘the number of golden fleeces is one, and the number of all other fleeces is ninety-nine’ would have (absurdly) an interpretation that does not imply ‘the total number of fleeces is one hundred.’

The last chance for denying ontological commitment of the existential quantifier is to maintain that it is unequivocally lightweight. I offer only a simple argument against this view: ‘There are Fs’ has the same truth value as ‘there *really* are Fs’, and the latter entails the existence of Fs. So, the former does too. It is clear that ‘really’ is added to remove conversational suggestions that the speaker is lying, jesting, speaking sarcastically, or otherwise non-literally. The word ‘really’ does not change the truth value of a sentence to which it is added.

An argument that is given in some version by Thomas Hofweber, Steven Schiffer, and Kit Fine, I call the triviality objection. The argument is roughly that existentially quantified claims are trivial but the related ontological claims are not trivial, so the former do not imply the

latter. Fine takes *following obviously from evident facts* to be sufficient for triviality. For example, ‘there is a number’ follows trivially from the obvious truth: ‘there is an even prime number.’ However, ‘numbers exist’ is a substantive metaphysical assertion.⁷

This argument, I believe, begs the question. No one who believes that quantifier claims are ontological claims will accept that ‘there is a number’ is any more trivial by Fine’s condition than ‘numbers exist.’ The former does follow from the evident fact that ‘there is an even prime number,’ but the latter follows from the equally evident fact (in the eyes of Fine’s opponent) that ‘an even prime number exists.’ It cannot be assumed that the quantificational claim is not equivalent to the ontological claim—not in an argument propounded to establish a difference between quantificational claims and their ontological analogues. It is open to Fine’s opponent to decide whether the ontological claims are trivial or whether the quantificational claims are non-trivial. The important point here is to notice that the modest claim of chapter one—that a circumscribed group of sentences are ontologically committed—could be easily expanded. By virtue of implying sentences in the group of committed sentences (the fundamental and simple existentially quantified claims or FSEQs), a great number of other sentences bear ontological commitments.

Fine’s next argument is that ‘natural numbers exist’ does not imply ‘integers exist,’ however, ‘there is a natural number’ implies that ‘there is an integer.’⁸ It follows that the ontological claims are not equivalent to quantificational claims. In response, I argue that the realist claims are ambiguous and that on one reading (where they are equivalent to the existentially quantified claims) ‘natural numbers exist’ does imply ‘integers exist.’ On the other

⁷ Kit Fine, ‘The Question of Ontology’ in *Metametaphysics*, ed. David Chalmers, (New York: Oxford University Press, 2009): 158

⁸ *Ibid.*, 166.

reading, where the former does not imply the latter, the two are not FSEQs but more complex quantificational claims (that are not implied by the simple existentially quantified ones).

Finally, in the last section I argue that existential generalizations from identity statements of the form ' $c = c$ ' are FSEQs since they are fundamentally represented as simple, existentially quantified statements and are equivalent to numerical claims. Consequently they imply the existence of c . In the next chapter I defend existential generalization from its primary challenger, positive free logic.

Chapter Two Summary: Positive Free Logic and Supervaluational Semantics

A positive free logic is one in which sentences containing non-referring terms can be true. There are two main varieties of positive free logic (PFL): bivalent PFL and supervaluational semantics (SVS) in which atomic sentences with empty names are neither true nor false. SVS counts as a PFL since some sentences, like the classical tautologies, are true whether or not they contain empty names. The semantics of bivalent PFL is most clearly understood as having two domains, an inner domain and an outer domain. The quantifiers range over the inner domain but the elements of the outer domain can have properties and stand in relations.

I argue that this kind of logical system is inadequate to capture the first order fragment of natural language since it fails to make valid one of the most basic, prototypically valid forms of inference. What I call the prototype schema—All A are B and x is A, so x is B—has invalid instances on bivalent PFL. I present what amounts to an argument from authority in defense of the schema. In every period (at least) since Aristotle, writers on logic have endorsed the validity of prototype arguments. According to the defender of PFL all of these authors, despite attempting to present and describe valid arguments, have been mistaken about such a simple

argument form or else have been suppressing premises required for the validity of prototype arguments.

It might seem possible to save the prototype schema on PFL by appealing to ambiguous quantifiers. I give another argument against the plausibility of that response. It is similar to the argument in chapter one against ambiguous quantifiers, but not exactly the same. The argument is that if the universal quantifier is ambiguous then so is the existential quantifier. However, existentially quantified sentences are used in sentences identical to numerical claims and those fail some tests of ambiguity. For example, if ‘number’ is ambiguous then it would be surprising if the various senses of number could be sensibly added to each other; however, it seems that purported cases of heavy and light numbers can be easily added and subtracted. Also, ‘the number of’ passes the conjunction reduction test. For example, conjuncts with distinct uses of the same term can be reduced in a way that leaves them, if sensible at all, only so in a way with a clever literary ring. For example, ‘upon viewing the picture of the cathedral, the point of focus became clear to her and the point of using buttresses became clear to her’ reduces to ‘the point of focus and of using buttresses became clear to her.’ The reduced sentence exhibits zeugma, but would not if ‘point’ were not ambiguous. Purported heavy and light number conjunctions do not reduce to any literarily interesting sentences; consider ‘the number of Mars’ moons and of Tatooine’s suns is two.’

Supervaluational semantics allows some sentences with non-referring names to be true; so, it rejects unrestricted existential generalization and avoids the troublesome consequence that simple existence claims are knowable a priori. SVS has other advantages as well. It preserves the classical logical truths while accommodating intuitions that atomic sentences with empty

names are neither true nor false. The idea, which originated with Bas van Fraassen,⁹ is to complete interpretations with empty names by arbitrarily assigning domain elements to them. If a sentence is true (false) on every completion of an interpretation then the sentence is true (false) on the interpretation. If the sentence is true on some completions and false on others then the sentence is neither true nor false on the interpretation. So on an interpretation where ‘Pegasus’ has no referent, ‘Pegasus has wings’ is neither true nor false, but ‘Either Pegasus has wings or not’ is true.

In addition to the advantages claimed for SVS in the preceding paragraph, there are others. The SVS approach can be applied to resolve several other philosophical problems as well. Perhaps most notably, a similar SVS strategy has been employed to save classical logic while still allowing that sentences containing borderline cases or vague predicates are truth-valueless. Instead of completions that assign domain elements to all names, SVS for vagueness has precisifications under which every predicate partitions the domain. Truth on an interpretation is truth on all precisifications of it. Also, in chapter two, I mention several other uses to which a supervaluational semantics has been put. I believe the appeal of SVS comes from its diversity of applications. I call this appeal to application the ‘utility defense.’

There are several well-known criticisms of SVS. After saying why my prototype schema argument against PFL is unconvincing as a criticism against SVS, I discuss several of them. First, SVS has an oddity: all versions of it permit true disjunctions without true disjuncts! Some versions of it have a corresponding problem for the existential quantifier; SVS’s resolution of the Sorites paradox relies on there being true existential generalizations without any true instances. True disjunctions without true disjuncts is odd, however this feature of SVS is its great

⁹ Bas C. Van Fraassen, “Singular Terms, Truthvalue Gaps and Free Logic,” *Journal of Philosophy*, 63 (1966): 481-95.

advantage. Saving the classical tautologies like the law of excluded middle (LEM) while at the same time rejecting bivalence requires that ‘True(A \vee B)’ not imply ‘True A \vee True B’ otherwise the truth of an instance of $P \vee \sim P$ would imply either the truth of P or the truth of $\sim P$. I propose that using the oddity as a criticism of SVS amounts to begging the question. It is possible that there be an argument that LEM implies bivalence; I consider several in chapter two, and show that they all rely implicitly on bivalence.

The best criticism of SVS is not to be found in its strange consequences but in its counterintuitive conception of truth. According to SVS, the truth of a sentence does not depend on a correspondence between the language and the world, but rather on a set of possible changes to the language!¹⁰ Bencivenga defends this counterfactual conception of truth by appeal to an analogy: we use practical experiments to ascertain the truth of atomic sentences so it is appropriate to use ‘mental experiments’ (completions) to ascertain the truth of sentences where practical experiments are not possible (as in cases of reference failure). I argue in response that the dissimilarities between cases of successful reference and unsuccessful reference weaken the analogy.

If this simple criticism is successful of the SVS approach, it remains to give a criticism of SVS’s argument for that approach. In response to the utility argument (that SVS saves the classical logical truths while rejecting bivalence) I present a way for neutral free logic to account for the intuitive appeal of the classical logical truths. All of the classical logical truths have obviously corresponding, obviously valid inferential forms and the validity of those forms is preserved by a neutral free logic.

Though I believe this is sufficient to put the intuitive appeal of NFL on par with that of SVS for names—which is my ultimate goal—there is another component of the utility defense

¹⁰This sort of criticism appears in Lehmann’s “More free logic,” 233.

that is untouched by the availability of these inferential forms of classical logical truths: some versions of SVS (SVS for vagueness, for example) preserve some apparent contingent truths (or ‘penumbral truths’). For example, the conditional ‘if George Bush is tall then Barack Obama is tall’ comes out true on SVS for vagueness, as it intuitively should, despite having a neutral antecedent and a neutral consequent. I argue that the intuitive appeal of *contingent* truths does not carry much weight when evaluating a logical system, and furthermore that SVS *loses* contingent truths sufficient to outweigh the gain from preserving penumbral truths. Specifically, the problems that SVS are designed to address turn out false on SVS—consider ‘The predicate ‘tall’ is vague.’ On no precisification of ‘tall’ does that sentence come out true. Also, some language is *intentionally* imprecise, so it is absurd that on SVS for vagueness it comes out true that ‘there is an exact (to infinitely many decimal points) distance from me within which someone must be in order to be in my general area.’ These contingent absurdities are not, on their own, a reason to reject SVS; however, they are sufficient to blunt the force of the argument that SVS succeeds because it preserves some key contingent truths.

If the arguments of chapter two are successful, then there are no serious contenders for rejecting existential generalization. The only remaining way to avoid the consequence of the master argument above is to reject the logical truth of ‘ $a = a$.’ There are two main ways to do it. One might say that the sentence has neutral instances or say that it has false instances. Negative free logic takes the second option. The primary reason for choosing the initially appalling second option is to retain bivalence. I turn to that issue in chapter three.

Chapter Three Summary: Negative Free Logic and Bivalence

A negative free logic (NgFL) regards atomic sentences with empty names as false. All of the classical tautologies are preserved and the semantics is bivalent. Existential generalization is

restricted in order to prevent deriving false existential claims such as ‘something is not identical with itself’ from oddly true (for Negative Free Logic) sentences like ‘it is not the case that Santa is identical to Santa.’ The main proponents of negative free logic are Rolf Schock, Tyler Burge and Richard Sainsbury. If there is any substantial disagreement among them about the particulars of negative free semantics, it is over a name scope device. Sainsbury stops short of employing one but mentions a name scope device and later suggests a criticism to which it would provide a response.¹¹ I argue that allowing names to take different scope relative to negation introduces more problems than it resolves; for one thing, it absurdly makes denials of three place relations, like ‘Al did not tell Barb about Cal,’ eight-ways ambiguous.

Even for negative free logic without name scope ambiguities there are oddities. I present three criticisms. None of them are conclusive, but they do together seem to require a justification from the negative free logician. The first is a problem mentioned by Sainsbury: some seemingly equivalent statements could have different truth values. ‘A is not identical to B’ and ‘A is distinct from B’ have different truth values if either A or B fails to refer and if ‘is distinct from’ is a simple relation. The second objection is that a close relative of the prototype argument (of chapter two) comes out invalid on NgFL: $\forall(\sim Ax \rightarrow Bx), \sim Ap, \therefore Bp$. Any model on which p is empty and the non As are a subset of the Bs, gives the intuitively valid argument all true premises and a false conclusion. The third objection is that NgFL loses another obviously valid inference—this one involving identity. On NgFL, ‘everything identical with A is identical with B’ no longer validly implies that ‘A is identical to B’.

¹¹ In *Reference without Referents* (New York, Oxford, 2005), Sainsbury mentions the name scope alternative on page 70 and later, on page 198 suggests the troublesome sentence ‘Vulcan is not more than 1000 miles in diameter.’ The sentence is true for him but seems false if uttered by Le Verrier as reporting a maximum size for a hypothesized but non-existent planet. A name scope ambiguity would solve the problem, though Sainsbury does not appeal to one.

Accepting these oddities requires some justification, and the negative free logicians have offered some. After reviewing a few of those justifications, I conclude that they all come down, in one way or another, to bivalence. If there are no good reasons for accepting bivalence then there are no good reasons to prefer negative free logic to neutral free logic. It cannot be just obviously absurd to deny bivalence since everyone denies it for some class of objects and accepts it for another. The question is where to draw the boundary between the classes. The placement of that boundary is not so obvious that someone, in defense of NgFL, can rely on the absurdity of not placing it precisely her way.

Bivalence is the view that all truth bearers are either true or false. That is distinct from the Law of Excluded Middle (LEM), which holds in any logical system in which $P \vee \sim P$ is a theorem. There is one well-known argument for Bivalence—Timothy Williamson's argument.¹² Williamson declares that his argument is not a defense of bivalence for sentences with empty names. However, I consider it as if it were since I know of no other arguments for bivalence of such sentences.

Williamson argues for:

(B) If an utterance U says that P, then U is either true or false.

His argument, which can be presented in a number of ways, relies the following two principles;

(T) If U says that P then U is true iff P.

(F) If U says that P then U is false iff $\sim P$.

Williamson presents a *reductio ad absurdum*. If there is a counter example to (B) then U says that P and is neither true nor false. By (T) it follows that $\sim P$ and by (F) it follows that $\sim\sim P$. So (B) cannot be consistently denied.

¹²Williamson, *Vagueness*, 188.

It is challenging to determine exactly what Williamson intends by ‘says’ and it is possible to criticize the argument on those grounds; however, for the sake of argument I suppose that utterances with empty names say something (even though Williamson never claims they do) and find other grounds to criticize the argument. After surveying a few criticisms of the argument I give reason to think that the important criticism is rejection of LEM.

To see why, consider a schematic argument that is simpler than Williamson’s:

$$P \vee \sim P$$

$$P \leftrightarrow T(P)$$

$$\therefore T(P) \vee T(\sim P)$$

If the second premise (the T-schema) implies the intersubstitutability of P and T(P) then the argument is valid. The neutral free logician could not simply reject the T-schema and thereby comfortably avoid the statement of bivalence in the conclusion. Even though NFL does not accept the truth of every instance of the T-schema, the replacement of P by T(P) is truth preserving for NFL. If P is true then it contains no empty names. So P and T(P) have the same truth value. Since the connectives of NFL are truth-functional, T(P) can be substituted for P and the argument (...P...), \therefore (...T(P)...) is semantically valid. Consequently, the conclusion of the simple schematic argument follows on NFL from the first premise alone. So, for NFL, denial of bivalence requires denial of LEM.

It is worth noting that NFL’s ‘rejection’ of LEM is not an admission of false instances of it. Even if the conclusion of the simple schematic (valid) argument is false the premise is not. The falsity of the conclusion implies the neutrality of the premise.

For the remainder of the chapter, I turn my attention to LEM. I first consider defenses of it and then offer several criticisms of LEM. The first support for LEM comes from opponents of

intuitionist logic, which does not allow premise-free deductions of LEM instances. The argument, originally presented by Marcel Barzin and Alfred Errera in 1927, claims that denying LEM is contradictory.¹³ What follows here is a simplified version of the argument that makes minimal effort to obscure its error: if LEM is rejected then there is some P for which it is not the case that $P \vee \sim P$. So there is a P for which $\sim(P \vee \sim P)$. But that is just to accept that for some P, $\sim P \cdot \sim \sim P$, which is a contradiction. Alonzo Church's 1928 response is to the point;¹⁴ the argument is question begging. Supposing that denial of LEM requires acceptance of its negation assumes that either it or its negation is the case. However, one who rejects LEM need not accept its negation. She could, as NFL recommends, merely regard some instances of it as neutral rather than as having true negations (or equivalently: as being false).

The second defense of LEM is again presented as a problem for rejecting it. Because he mentions it in publication, I cite Hartry Field although the idea certainly predates him and he does not endorse it. If instances of LEM and other tautologies are dubious in cases of indeterminacy and reference failure then they are perpetually dubious since the threat of indeterminacy and reference failure are omnipresent. The result is that familiar tautologies and consequently, classical logic are useable for very few purposes.¹⁵ My response is that the sacrifice is minimal and we are warranted in requiring some minimal justification for instances of LEM before relying on them in argument—otherwise we could draw absurd conclusions, like that of the master argument in this introduction, to the effect that contingent objects can be known to exist a priori.

¹³ Marcel Barzin and Alfred Errera, "Sur la logique de M. Brouwer," *Académie Royale de Belgique, Bulletin* 13 (1927): 56–71.

¹⁴ Alonzo Church, "On the law of excluded middle," *Bulletin of the American Mathematical Society* 34, no. 1 (1928): 75–78.

¹⁵ Hartry Field, *Truth and the Absence of Fact* (New York: Oxford University Press, 2001): 287.

I then give three reasons to reject LEM. The first is to soften the audience and the second two are more in line with NFL's reason for rejecting LEM. The first comes again from Hartry Field.¹⁶ Finitely many instances of LEM and some benign premises about the beginning of Jerry Falwell's life absurdly imply that there is an exact (to arbitrary specificity) nanosecond during which Jerry Falwell was born. Since this is preposterous, at least one of the instances of LEM should be rejected. Since the semantics I present in chapter four makes no accommodation for LEM violations of this type, I move quickly to criticism number two.

Consider an instance of LEM: Sherlock has zero warts $\vee \sim$ (Sherlock has zero warts). Intuitively, since there is no Sherlock (and nothing even in fictional accounts of Sherlock about whether he has warts) there is no fact that could plausibly make either disjunct true. However, NgFL will respond that despite intuitions to the contrary, one of the disjuncts is true. My criticism takes the form of a dilemma: if 'Sherlock has zero warts' is false on NgFL then, strangely, it is not equivalent to 'Sherlock has no warts' which is true on NgFL if 'Sherlock' is empty. This sacrifices the connection between quantifier and number defended in chapter one. If 'Sherlock has zero warts' is true on NgFL then it must be because it is equivalent to 'Sherlock has no warts.' But then the following argument is not valid on NgFL:

Sherlock has zero warts.

Joe has one wart.

For all $a, b, x,$ and $y,$ if a has x warts, and b has y warts, and $x > y$ then $a \succ b.$ ¹⁷

So, Joe has more warts than Sherlock.

Since the conclusion is an atomic, binary relation with 'Sherlock' as a relatum, it is false on NgFL, even on models where the premises are all true. NgFL has few plausible alternatives to

¹⁶ Hartry Field, "No Fact of the Matter," *Australasian Journal of Philosophy* 81, no. 4, (2003): 458.

¹⁷ Where \succ means 'has more warts than.'

the incredible conclusion that the argument is invalid. So the best course of action is to deny the logical truth of at least one instance of LEM.

The next argument against LEM is metalinguistic. Consider two related sentences:

- ★ Something is such that it either has warts or it does not.
- ★★ Sherlock is such that either he has warts or he does not.

Since ★ is possibly untrue, ★★ is possibly untrue. In a world with nothing, ★ is untrue because there is not anything such that it either has friends or not. So it is not true that Sherlock is such that he either has friends or not. The published NgFL accounts at least all accept the premise that ★ is possibly untrue. So the real options for criticism are to claim that the possible untruth of ★★ is consistent with LEM or to reject the validity of the argument. There is no clear way forward for the first option so the second is the likely response. However, the only reason that I can see for rejecting the validity of the argument is a rejection of existential generalization. But the best reason going for rejection of existential generalization is NgFL, and the best reason going for NgFL is its salvation of classical tautologies—and of course, that justification cannot be given against an argument challenging the preservation of the classical tautologies.

If LEM and bivalence are without defense then so is NgFL. The only remaining route for avoiding the absurdity of the above master argument then is a neutral free logic. I now turn to an explanation and defense of the particular version of neutral free logic that I endorse.

Chapter Four Summary: Semantics for Neutral Free Logic

A semantic account for neutral free logic must decide how to assign truth values on the empty domain, how to handle identity statements, what tables to use for the connectives, what rules govern quantified statements, and how to define logical truth and logical consequence. In

chapter four, I consider each and defend my decisions. Then I present my semantics in technical detail and finally defend it from one criticism by another version of neutral free logic.

A semantics that allows the domain to be empty is *universally free or inclusive*. Since I know of no successful argument that the existence of something should be a *logical* truth, I present an inclusive logic. The question then is how to assign truth values on the empty domain. The standard approach to quantifiers is to make all universally quantified sentences true and all existentially quantified sentences false. There is an alternative. One could assign sentences with vacuous quantifiers the same value that they would have without the vacuous quantifier. This variation would have the bad consequence that some sentences with a leading existential quantifier would come out true on the empty domain, for example ‘something is such that nothing exists.’ The consequences of the standard approach are less than appealing too. ‘Everything is such that P’ would not logically imply P.

Perhaps that last point is not discomfoting enough to abandon the standard approach. In chapter four, I argue that no sentence should have a truth value on the empty domain. If successful, my arguments for that conclusion are sufficient to undermine both variations mentioned in the previous paragraph. The first point is a technical difficulty; a semantics that makes universally quantified sentences true on the empty domain must employ a device other than denotation and assignment functions (or beta-variants of denotations, or expansions of denotation functions) since no functions map to the empty set. Another, more interesting problem for making universally quantified sentences true on the empty domain is that it results in the loss of some intuitively valid arguments. Here is one:

Everyone, despite the fact that some people are unselfish, is self-interested.

So, some people are unselfish and everyone is self-interested.

Limiting the domain to people, the argument is most naturally rendered:

$$\forall x((\exists yUy) \cdot Sx)$$

$$\therefore \exists yUy \cdot \forall xSx$$

A glance at the leading quantifiers reveals at once why the standard approach to the empty domain invalidates the argument. Here is another valid argument lost by a standard inclusive logic:¹⁸

Anyone identical to Samuel Clemens is an author.

So Samuel Clemens is an author.

Because my interest lies with saving valid inferences rather than with saving logical truths (since logic is a guide for reasoning not a catalogue of facts), my approach to the empty domain is to use the standard semantic devices of denotation and assignment functions to assign truth values. Since on the empty domain there can be no denotation or assignment functions, on the empty domain there are no truth values. This accomplishes the goal that motivates an inclusive logic: ‘something exists’ should not be a logical truth. Furthermore, it preserves all and only the valid inferences of a neutral free logic that does not allow the empty domain. My approach could not validate new arguments since any arguments that are valid on a set of models are valid on that same set less one member. Also, it cannot invalidate any arguments because to do so there would need to be a model where the premises of an argument were true, but no premises are true on the empty domain.

One might object to my simple approach because it loses all logical truths. However, this should not be seen as a major catastrophe since indeterminacy and counterexamples to LEM like Field’s in Chapter Three threaten to undermine the logical truth of quantified claims anyhow.

¹⁸ It is possible to avoid this result by modifying the standard inclusive logic’s quantifier rules so that any sentence with an empty term is neutral. ‘Samuel Clemens’ would be empty if the domain were, so the argument would be valid. This is the approach taken by McKinsey.

One might object to my account because on it, one could not truly say that nothing exists, even though that is intuitively true on the empty model. In response, I argue that our intuitions about contingent truths on bizarre models should not weigh heavily enough to abandon seemingly valid arguments like those mentioned above.

The next decision that must be made by NFL is how to handle identity statements. The prevailing attitude for someone inclined to NFL should be that empty terms neutralize identity claims. However, one could take a play from NgFL and ensure that $\exists x(x = c)$ is false when c does not refer by making identity sentences false when at least one term does not refer. Intuitively, ‘something is Santa’ ought to be false when ‘Santa’ is empty. I think it is unfruitful to try to accommodate that intuition, but it is especially bad to accommodate it by making some instances of $a \neq a$ true! The obviously valid Samuel Clemens argument above (Anyone identical to Samuel Clemens is an author, so Samuel Clemens is an author) would have populated-domain counterexamples. On a model where ‘Samuel Clemens’ is empty, the argument would have a vacuously true premise, yet would also have an untrue conclusion.

A neutral semantics must also decide on the definitions for the connectives. A motivating factor in the acceptance of NFL is the conviction that a sentence’s truth-value is a function of the values of its constituents, and in the case of an empty name, a crucial constituent is missing. The result is a lack of truth value. So, the obvious choice of definitions is weak Kleene tables which are classical except that any neutral inputs result in neutral outputs.

Sainsbury, a defender of NgFL presents a difficulty for a logic with weak Kleene tables. One could always recover truth values from neutrality by means of an operator (*neg*) that functions similarly to the natural language phrase ‘it is not true that.’ It would take true inputs to false and

take false or neutral inputs to true.¹⁹ In response, I argue that there is no such object language operator in natural language. The operator is metalinguistic. Since NFL is an object-level logic, the possibility of the meta-linguistic operator in English does not count against our choice of weak tables. I argue that Sainsbury's *neg* operator is metalinguistic (in natural language) by means of an analogy between it and quotation.

The similarities are three-fold: like quotation, *neg* is opaque to quantification. For example 'Joe said "Pegasus is something,"' does not imply, 'there is something such that Joe said "it is something."' Similarly, '[Neg] Pegasus is something,' does not imply, 'there is something such that [neg] it is something.' Also, like quotation, *neg* permits the introduction of novel words. Just as, 'Joe said "something is flarv,"' is possibly true, so is '[neg] something is flarv.' Finally, like quotation, *neg* would permit empty demonstratives. 'Joe said "that is something"' is possibly true, as is '[neg] that is something.' So in addition to its description as equivalent to the metalinguistic phrase 'it is not true that,' there is compelling reason to take the [neg] operator to be a metalinguistic device, rather than the natural language, object-level operator that would undermine the choice of weak Kleene tables.

The next concern is the semantic rule that governs quantifiers. As mentioned above, 'something is Pegasus' is intuitively false if 'Pegasus' is empty. One way to accommodate that intuition is to make quantifiers bivalent. One version of bivalent quantifiers is proposed by Lehmann in 2002.²⁰

$\forall xP$ is true if every value of x makes P true, otherwise it is false.

$\exists xP$ is true if some value of x makes P true, otherwise it is false.

¹⁹ Sainsbury, *Reference without Referents*, 67.

²⁰ Lehmann, "More free logic," 234.

The problems with these rules all seem to stem from the fact that they do not preserve the equivalences between \forall and $\sim\exists$ and between $\sim\forall$ and \exists . To mention just one problem, on this account, ‘there is not anything that is taller than Joe’ absurdly does not validly imply ‘everything is not taller than Joe.’ If ‘Joe’ is empty, the premise is true and the conclusion is false.

One way bivalent quantifiers and the quantifier negation equivalences can be saved is by making the existential rule:

$\exists xP$ is false if some value of x makes P false, otherwise it is true.

This method of saving the quantifier negation equivalences has the disastrous consequence of making ‘something is Pegasus’ true if ‘Pegasus’ is empty. This of course, fails to honor the intuition that suggested bivalent quantifiers in the first place.

So, three-valued quantifiers are best. In 1960, Timothy Smiley proposed roughly the following rules:²¹

$\forall xP$ is true if every value of x makes P true, and is false if some value of x makes P false.

$\exists xP$ is true if some value of x makes P true, and is false if all values of x make P false.

These rules preserve the quantifier negation equivalences. However, they falter on the empty domain. They preserve the quantifier negation equivalences except when the domain has no members. In that case, the universal quantifier rule is vacuously satisfied and sentences like ‘everything is identical to Pegasus’ come out true. So, some modification is required. I formulate my similar rules in terms extensions of the language and denotation function, so that if there is no denotation function (this happens just in case there is an empty target set or domain) there is no truth value assigned. An alternative that preserves some truths on the empty domain without accepting the truth of ‘everything is identical to Pegasus,’ is to include a clause in the

²¹ Timothy Smiley, “Sense without denotation,” *Analysis* 20, no. 6 (1960): 126.

quantifier rules that explicitly neutralizes any sentence with an empty term; this is the approach taken by McKinsey.

The decisions made so far lead to the loss of logical truths; they also lead to the loss of some classical syntactic derivation techniques. The semantics given invalidates the (classically valid) arguments wherein the conclusion contains components not present in the premises. This means that some of the derivation rules that are sound according to classical semantics are unsound. Specifically, NFL loses all of the rules that allow the introduction of novel terms (e.g., addition, zero premise deductions, universal instantiation).

One loss might be regarded as especially troublesome. Conditional proof seems to be the cornerstone of natural deduction systems. So, since NFL rejects conditional proof, perhaps no natural deduction system is possible for it. However, if it is possible to give a natural deduction system for a semantics without a conditional (as it seems to be, given the possibility of dispensing with the conditional altogether) then conditional proof is not a requirement. Furthermore, a *CP-like* rule can be employed. It requires only existence assertions for any constants appearing in the premises, or that the constants appear in earlier (unassumed) lines of a derivation. This is not too drastic a departure from ordinary reasoning.

In chapter four, I also consider the loss of other derivation rules. Of course, the loss of these derivation rules is a positive thing from the point of view of NFL since those rules represent semantically invalid inferences.

After presenting the technical semantic rules that achieve the desiderata suggested by the forgoing considerations, I address one response to the master argument that has not been mentioned yet: what we know *a priori* are hedged claims. We do not know *a priori* that Pegasus is self-identical but rather some kind of conditional to the effect that *if Pegasus exists* then

Pegasus is self-identical. If this approach is to do the trick (of explaining our acceptance of something like classical logical truths and at the same time avoiding the absurd consequence that we know a priori some contingent existence claims) then the hedged claims must be true whether or not the antecedent is.

A neutral free logic like the one I defend cannot admit hedging like this; because of the weak Kleene tables, empty names are like poison. An empty name in the antecedent of an attempted hedge neutralizes the sentence. It seems as if strong Kleene tables and the material conditional might be well-suited for hedging if untrue existence claims were false. Strong Kleene tables are classical except that neutral inputs are treated as if they were unknown classical values.²² That is, a neutral input only results in a neutral output if you could not establish the classical truth value of the output sentence without knowing whether the input was true or false. For example, on strong Kleene tables, a conditional with a false antecedent and a neutral consequent is true since you could tell on classical tables that the sentence was true before discovering the value of the consequent. On the other hand, according to strong Kleene tables, a conditional sentence with a true antecedent and a neutral consequent is neutral since we would have to know the value of the consequent to assign a classical value.

Unfortunately, hedging is not so simple. For one thing, it is intuitively possible to half-hedge or partially hedge a claim, so that it implies the existence of one thing but not another. For example, ‘if Santa exists then Santa lives with Mrs. Claus,’ does not seem to require that Santa exists but does seem to require that Mrs. Claus does. However, if the conditional is material then the (seemingly) half-hedged sentence does not validly imply the existence of Mrs. Claus; the

²² This epistemic description of the strong Kleene tables is not a fundamental feature of them. I mention it only as a way of describing the tables succinctly without reproducing them.

false antecedent makes the sentence true whether or not the consequent is neutralized by a different empty name. It follows that the simple material conditional is too weak to hedge.

If the conditional used for hedging is not material then it has some strange features. I give an argument that a (non-material) hedging conditional has to give up at least one of the following (1) necessitation of it by strict implication, (2) validity of exportation for it, or (3) its untruth if the antecedent is true and the consequent is untrue. Furthermore, if the hedging conditional is not material then it is difficult to maintain that contraposition is valid for it in the face of counterexamples like: ‘If Santa exists then Santa does not wear a green suit.’ Just try to justify the truth of the contrapositive—‘if Santa wears a green suit then Santa does not exist’—without appealing to the truth table for the classical material conditional.

James Pryor in an unpublished addendum to a 2006 paper²³ tries to save the material conditional for hedging by using it within the scope of a universal quantifier. To formalize, as Pryor would, a hedged sentence like ‘If Jack exists then Jack is taller than Jiho’ replace all instances of ‘Jack’ in the consequent with a variable, set the variable equal to Jack in the antecedent and then bind the result with a universal quantifier. So for the sample, the result would be: ‘ $\forall x[(Jack = x) \rightarrow (x < Jiho)]$.’ Call the result ‘Jack-hedged.’ For Pryor the trivalent universal quantifier takes the lowest value of all assignments. So the formalized sentence implies that Jiho exists without implying that Jack does, and the material conditional handles half-hedging!

I argue that Pryor’s attempt is ultimately unsuccessful. He claims of his semantics that an α -hedged sentence will never entail that α exists, but that an α -hedged, but not β -hedged, sentence might still entail that β exists. This would mean that hedging is always successful and

²³ James Pryor, “More on hyper reliability and A priority,” Unpublished (2006) (<http://www.jimpryor.net/research/papers/More-Hyper.pdf>).

that partial hedging is possible. However, on Pryor's semantics, because of his treatment of the empty domain, partial hedging is impossible. There is a simple fix for retaining the semantics; it is to rule out the empty domain. The problem with that 'fix' is that it makes hedging sometimes unsuccessful.

Because on Pryor's semantics all universally quantified sentences and no existentially quantified sentences are true on the empty domain, no α -hedged sentence implies the existence of anything. So partial hedging is not possible. If we ignore the empty domain then a sentence like 'if Jack exists then he is identical to Kyle but not to Lin,' will not be Jack-hedged, even if it is formalized in the prescribed way: ' $\forall x[(Jack = x) \rightarrow (x = Kyle \cdot x \neq Lin)]$.' To see why, suppose 'Jack' is empty. If either 'Kyle' or 'Lin' is empty then the consequent is neutral for some substitution (or false if Lin is the only domain element), and if neither 'Kyle' nor 'Lin' is empty then the consequent is false for some substitution. So, the sentence cannot be true unless 'Jack exists' is.

The fourth chapter concludes with some remarks that are intended to explain away the intuition that we can hedge against non-existence. The two guiding intuitions—that we can hedge and partially hedge against non-existence—are accommodated by hedging with a description rather than the name. For example, on the neutral free semantics defended in chapter four, 'If 'Jack' has a referent then it is taller than Jiho,' implies the existence of Jiho without implying the existence of Jack. It also handles all of the trouble case that I present for hedging generally. I do not attempt to prove that there are *no* potentially troublesome cases for hedging in this way.

That completes the outline of the chapters to come. For the remainder of the introduction I sketch a defense of the fifth premise in the master argument above.

Closure of the A Priori

In this section I give two arguments for the fifth premise in the master argument (5). The first is a simple argument from two weaker principles. The second is an argument from a closure-like principle for knowledge.

The Simple Argument

The principle expressed by 5 can be defended by appeal to two more obviously acceptable principles. (Read \Box as ‘apriori knowable.’)

$$\star \quad \forall P \forall Q (\sim \Box P \rightarrow \sim \Box (P \cdot Q))$$

$$\star\star \quad \forall P \forall Q [(P \vDash Q, Q \vDash P, \Box P) \rightarrow \Box Q]$$

The first says that if P is not knowable a priori then the conjunction of P with Q is not knowable a priori. This claim could be defended with 5, but need not be so (circularly) defended. If one cannot know a proposition by mere consideration of it then surely, one cannot know that proposition *and more* by mere consideration. The second could again be defended with 5 but need not be. It says that if P and Q are logically equivalent and P is a priori knowable then so is Q. It may be that $\star\star$ is subject to criticisms similar to those of 5, but it also seems to be open to more possibilities for justification. For one simple (but unsuccessful) example, one might argue that if P and Q are logically equivalent then they express the same proposition, so that knowledge of P *just is* knowledge of Q.²⁴

²⁴ Difficulties for this simple argument were pointed out to me by Eric Hiddleston and Larry Powers (for example, knowing that $1+1=2$ is not knowing the Goldbach conjecture even if they are logically equivalent) and I do not endorse the argument; but it does seem to illustrate that the possibilities for defending $\star\star$ outnumber those for 5.

To see that \star and $\star\star$ together imply 5, suppose that P and Q are a counterexample to 5, so that $\Box P, P \vDash Q$ and $\sim \Box Q$. From $\sim \Box Q$ and \star it follows that $\sim \Box (Q \cdot P)$. However, if $P \vDash Q$ then $P \vDash (Q \cdot P)$ and, by $\star\star$ and $\Box P$, we have $\Box (Q \cdot P)$ which is a contradiction.²⁵

An Argument from Knowledge Closure

What follows is an argument for 5 from (Positional) Knowledge Closure. Here again is the version of closure of a priori that I will argue for:

5. What is logically entailed by a priori knowable truths is a priori knowable.

Restated in the symbols introduced in this section:

$$5'. \quad \forall P \forall Q [(\Box P, P \vDash Q) \rightarrow \Box Q]$$

Since ' $\Box P$ ' represents 'P is knowable a priori,' I must say what it means to know something a priori, which is a special case of knowing something. For S to know P three conditions must be met: (a) P is true, (b) S has sufficient justification for P, and (c) S believes P based on that justification. By 'sufficient' in (b) I mean 'enough for knowledge and of the (non-lucky) kind that avoids Gettier-type counterexamples.' The circularity should not be a problem. I will also avoid some inconsequential complication with (c) by not giving a substantive account of just when belief is based on justification.

The distinction between propositional and doxastic justification will be useful so I introduce it here. P is *propositionally* justified if there are propositions available to S that would justify S in believing P if she did so based on those propositions. S is *doxastically* justified in

²⁵ This reliance on proof by contradiction (RAA) and my later reliance on conditional proof (CP) are unproblematic from the point of view of most neutral free logics, since the conclusions contain no names—only quantified variables. On the semantics I defend in Chapter Four, because of its treatment of the empty domain, even when no names occur, RAA and CP are unsound. Still this RAA and the later CP are acceptable because it is harmless to include as a contingent but trivial premise that 'something exists' (which ensures that the domain is non-empty and that the arguments are valid).

believing P if P is propositionally justified and S believes P on the basis of that justification. Roughly, (b) is the requirement that P be sufficiently propositionally justified, and (b) and (c) together is the requirement that S be doxastically justified in P.

The requirements for a priori knowledge differ explicitly only in condition (b), but since (c) makes reference to the justification in (b) we should think of it as changing also. S knows P a priori just in case (a) P is true, (b') S has sufficient non-empirical justification for P, and (c') S believes P based on that justification. Now 5' can be restated,

$$5''. \quad \forall P \forall Q [(\diamond \exists x(a, b', c')P, P \vDash Q) \rightarrow \diamond \exists x(a, b', c')Q]$$

The unusual modal construction in the antecedent should be read 'there exists a person x who satisfies conditions a, b', and c' with respect to P, and similarly for the consequent. Here is the version of knowledge closure I will use to defend 5'':

(KC) Necessarily, for all propositions P and Q and people x, if x knows P and P entails Q then x is in a position to know Q.

If being in a position to know a proposition is understood as having propositional justification for it, then (KC) can be rewritten in the notation of 5'' as,

$$(KC'') \quad \Box \forall P \forall Q \forall x [(x(a, b, c)P, P \vDash Q) \rightarrow x(b)Q]$$

Now I give the argument that 5'' follows from (KC''). I start with an outline of the argument and then give a more thorough version.

The argument considers two cases: either knowing Q is a precondition for knowing P or it is not. For the outline, I consider the cases in the reverse order of the detailed argument. If knowing Q is not a precondition for knowing (a priori knowable) P but $P \vDash Q$ then P *itself* is a priori p-justification for Q (otherwise the knowledge closure principle would have possible

counterexamples). If knowing Q is a precondition for knowing P then one could not have a priori justification for P without having a priori justification for Q. If some empirical justification were required for knowing Q and one could not know P without knowing Q then, contrary to supposition, P would not be a priori knowable. What follows is the detailed argument with the order of the two cases reversed and with the holes plugged.

To see that 5" (or 5) follows from KC" (or (KC)), instantiate to arbitrary constants (here I just instantiate to the variables themselves to avoid confusion) and assume the antecedent of 5". So, at some world, x knows P a priori, and Q is entailed by P. There are two possibilities for Q; either it is a precondition for knowing P or it is not. Suppose it is a precondition.²⁶ In that case, x could not know P without propositional justification (p-justification) for Q. P could be true without p-justification for Q, and P could be believed without p-justification for Q, so p-justification for P is unobtainable without p-justification for Q (if Q is a precondition). Since by supposition x has sufficient *non-empirical* p-justification for P, x must have sufficient *non-empirical* p-justification for Q (since sufficient justification for P requires sufficient justification for Q).²⁷ If it is possible for x to have non-empirical p-justification for Q, while Q is true, then it is possible for someone (probably S) to base a belief in Q on that justification. There are three kinds of potential counterexample. S might not be able to properly base a belief in Q because of (i) a defeater, because of (ii) the nature of the proposition, or because of (iii) the finitude of S.

On (i), if some necessary defeater makes it impossible for anyone to properly base a belief then there will not be sufficient non-empirical justification for Q; it will always be

²⁶ Martin Davies, in "Externalism, architecturalism, and epistemic warrant," in *Knowing Our Own Minds*, eds. C. Wright, B. Smith & C. Macdonald (New York: Oxford University Press, 2000): p. 25, offers this account of preconditions: Q is a precondition for P if Q can be shown to be a necessary condition for P by 'salient theoretical considerations.' He argues that warrant does not transmit to preconditions. Warrant is something like propositional justification.

²⁷ See Michael McKinsey, "Transmission of warrant and closure of apriority," in *New Essays on Semantic Externalism and Self-Knowledge*, ed. Susana Nuccetelli (Cambridge, MA: MIT Press, 2003): 103,4 for the original and clearer version of this argument.

defeated. On (ii), if the proposition is such that no one could believe it on the basis of its evidence then there will not be sufficient a priori justification for it. Some self-referential statements might be counterexamples; consider, ‘this statement cannot be properly believed true on the basis of evidence.’ If it can be believed then it is nakedly false, but that would make it impossible to believe. So it cannot be believed and as a result it is true. That argument is a priori. We can, I think, safely ignore counterexamples like this because omitting self-referential sentences from the domain of the quantifiers in 5" and (KC") does not diminish their applicability to the simple sentences needed for the master argument to go through. On (iii), if a reasoning deficiency or the short lifespan of S prevents her basing a belief in Q then, since such limitations are merely contingent, there will be a possibility wherein S avoids the limitation. So the consequent of 5" is satisfied, on the suppositions that the antecedent is satisfied and that knowing Q is a precondition for knowing P.²⁸

The other alternative is that Q is not a precondition for P. Then P, given that $P \models Q$, provides sufficient p-justification for Q, otherwise there would be possible violations of (KC"). Suppose that x satisfies (a, b', c') with respect to P and that satisfying (b) with respect to Q is not a precondition of satisfying (b) with respect to P. Further suppose that P is not sufficient justification for Q, then there will be scenarios (or worlds) where P is known and there is no other evidence (p-justification) for Q, in which case (b) is unsatisfied with respect to Q (so one could fail to be in position to know Q, which contradicts (KC)). Since P is sufficient p-justification for Q and by supposition x has sufficient non-empirical justification for P, x has sufficient justification for Q that is non-empirical. It is possible to believe Q on the basis of that

²⁸ This reliance on conditional proof is unproblematic for NFL. See footnote 25.

justification (the potential counterexamples of the previous paragraph can again be safely ignored), so the consequent of 5" is satisfied.

So it seems that 5 follows from (KC). It remains only to defend (KC).

Knowledge Closure

As David and Warfield point out, (KC) is not really a closure principle, since there is no property that is closed over entailment.²⁹ It is closely related to but weaker than a (genuine) closure principle for justification. I will consider criticisms of (KC) and of a stronger principle for justification,

(CPJ) For all propositions P and Q and people x, if P is propositionally justified for x and P entails Q then Q is propositionally justified for x.

Since (CPJ) implies (KC) a defense of (CPJ) is a defense of (KC),³⁰ even though criticisms of (CPJ) are not always criticisms of (KC). I will consider three objections to principles like (KC): The Easy Justification Objection, The Long Chain Doubt Objection, and Harman's Objection.

Easy Justification

David and Warfield object to (KC) on the grounds that it makes it too easy to be positioned to know. They say it is implausible that subject S be in a position to know an entailed proposition Q if S does not even believe that Q is entailed by P. The problem is especially striking where Q is a logical truth since it is entailed by every proposition. Anyone who knows anything is positioned to know all logical truths.³¹

It is tempting to soften the blow of this objection by appeal to NFL's dearth of logical truths. There are two reasons why it is not inappropriate. (a) I am using (KC) in an argument for

²⁹ Marian David and Ted A. Warfield, "Knowledge-Closure and Skepticism," in *Epistemology: New Essays*, ed. Quentin Smith (New York, Oxford, 2008): 171.

³⁰ This reliance on closure of 'defense' under implication might seem like question-begging but denying what I say here because of a denial of closure of justification under implication is almost a *reductio ad absurdum* of that stance.

³¹ David and Warfield, "Knowledge-Closure and Skepticism," 171.

NFL, so I should not appeal to NFL to defend it. Furthermore, (b) the logical truths lost in NFL can be recovered as valid inferences (see Chapter Two). So if being positioned to know all logical truths is strange then it is equally strange to be positioned to know them all in their new guise as entailments.

Here is the best response presented in steps. Step one: the objection is not persuasive against cases wherein an inference from P to Q is obvious but not believed, e.g., S is clearly in a position to know Q if it follows from P by a single application of Modus Tollens, even if S has formed no beliefs about the rules of classical logic. So a weaker rule than (KC) is unobjectionable; call it the ‘easy entailment version.’ Best response, step two: Since any entailment can be broken into smaller, easy entailments, repeated application of the easy entailment version of the rule delivers the seemingly (but ultimately not) absurd result that all are positioned to know all (including disbelieved) entailments of known propositions. Thus, (KC) can be true despite this strange consequence.

Recall that my use of (KC) takes ‘positioned to know’ to mean ‘has propositional justification for,’ which requires only that there be propositions available to the subject that would provide doxastic justification if properly used as a basis for belief. My response to the objection is that the propositions like ‘P validly implies Q’ are available to subjects in that they could be deduced by a simple chain of easy entailments. It might be objected to this response that while each easy step is justified, taken together they need not be.

Long-Chain Doubt

Joshua Schechter criticizes a principle similar to the following closure principle for justified belief:³²

³² Joshua Schechter, ‘Rational Self-Doubt and the Failure of Closure,’ *Philosophical Studies* 163, No. 2 (2013).

(CJB) If someone has a justified belief that P, bases a belief in Q on a competent deduction from P, and retains justified belief that P then that person has a justified belief that Q.

This principle is not the same as (CPJ); it is a principle of closure over competent deduction rather than entailment. However, the antecedent of (CJB) implies the antecedent of (CPJ) and the consequent of (CJB) implies the consequent of (CPJ). So, a counterexample to (CJB) is a counterexample to (CPJ) if the consequent of (CJB) is false for the right reason—specifically, if S (of the counterexample) fails to have a justified belief that Q *because Q lacks propositional justification*.

Schechter points out that someone might competently deduce Q from P by a long chain of simple deductions and, by virtue of justifiably doubting the reliability of her reasoning, come to have a defeater for the justification of Q. If it is true that the specter of incompetence over long deductions undermines the justification for Q and not merely one's justified belief that Q, then we have a criticism of (CPJ).

Despite initial appearances, this is not really a criticism of (CPJ). The self-doubt makes it difficult to form beliefs on the basis of our deduction; it does not undermine the propositional justification of Q, which is either all or part³³ of our justification for P—our *deduction* of Q from P is not our propositional justification for Q. To see this, notice that repeating the simple steps of the single premise deduction reduces doubt without providing new propositional justification for Q. S's confidence that she performed the deduction properly increases, which makes it easier to base a belief in Q on the deduction. However, repetition of the proof provides no novel propositional justification for Q. Since self-doubt can be reduced (and eliminated) without novel propositional justification for Q, self-doubt is not a challenge to propositional justification for Q.

³³ If Q is not a precondition of P then all p-justification of P could be p-justification for Q. If Q is a precondition for P then justification for Q is part of x's justification for P.

Schechter considers a *level confusion* response to his objection; S can be justified in believing Q without being justified in believing that she is justified. He credits this general response to William Alston and credits Timothy Williamson with applying it to defense of closure principles.³⁴ That response goes like this: evidence of fallibility does not undermine S's belief in Q but rather it undermines S's belief that her belief in Q is justified. I do not believe that I am just giving that response, or saying something that depends on its being a good response. I am not claiming that one can be doxastically justified without being justified that one is justified; rather, I am saying that doubting one's proof does not undermine one's propositional justification.

Harman's Objection

In *Change In View*, Harman criticizes the Logical Implication Principle,³⁵

(LI) The fact that one's view logically implies P can be a reason to accept P.

Strangely, he uses 'can' when he means 'is always a reason to accept P.' His criticism can be slightly modified and applied to (CPJ). Suppose that Mary believes that if she looks in the closet she will see a box of Cheerios. Further suppose that Mary looks in the closet and does not see a box of Cheerios. The justified proposition P', *Mary is looking into the closet and if Mary looks in the closet she will see a box of Cheerios*, does not give Mary a reason for believing that, Q', *she sees a box of Cheerios*. Rather upon seeing the closet lacking Cheerios, she should give up the conjunctive proposition. Notice this is no criticism of (CJB) because of its clause: 'and retains justified belief that P.'

³⁴ William P. Alston, "Level Confusions in Epistemology," *Midwest Studies in Philosophy* 5 (1980). Williamson's contribution is unpublished.

³⁵ Gilbert Harman, *Change In View* (Cambridge, MIT Press, 1986), 11.

Since *Mary sees a box of Cheerios* does not seem to be a precondition for P' the argument cannot be made that justification for Q' is required in order for the antecedent to be satisfied.

In response, note that (CPJ) is a synchronic principle. It does not say that if P is justified for x at some time and P entails Q then at any future time, Q is justified for x. Harman's criticism involves justification at two times. Once Mary sees that there are no Cheerios in the closet she no longer has propositional justification for P', though P' was justified prior to the viewing (by supposition). So, at no time is there a counter-example to (CPJ).

So the knowledge closure principle survives several different sorts of challenge. If that principle is correct then the closure of a priori knowability seems to follow. So, to avoid the a priori knowability of simple existence claims one must find fault with either premise one or four of the master argument. One way of rejecting 4 is to deny that existentially generalized claims imply existence claims. That option is considered in chapter one.

CHAPTER ONE: THE EXISTENTIAL QUANTIFIER AND ONTOLOGICAL COMMITMENT

The connection between the existential quantifier and existence claims has been, until recently, taken for granted. Developments in metaphysics have challenged the connection. I argue below that a class of existentially quantified sentences meeting three conditions do imply existence claims and consequently bear ontological commitments. The main argument in this chapter is largely a development of one of van Inwagen's arguments. I argue that sentences satisfying the three conditions are best represented by an objectual quantifier and are univocal. So, since some of them bear ontological commitments, all of them do. I consider two objections and finally argue that two sentence types that are easily derivable from commonly accepted claims satisfy the conditions and so bear ontological commitments—most importantly, existential generalizations from trivial identities entail existence claims. Premise (OC) in the introduction follows from the argument in this chapter.

Paraphrase and Ontology

Paraphrase plays a dual role in ontology. For some it is used as a way of discovering hidden ontological commitments; for others it is employed as a way of avoiding them. Suppose that sentence A, which seems not to imply the existence of Fs, is inter-paraphrasable with sentence B, which seems to imply the existence of Fs. This inference is common:

1. B is a paraphrase of A
2. B implies that Fs exist.
3. So, A implies that Fs exist.

It is exemplified in the following passage from David Lewis's *Counterfactuals*:

It is uncontroversially true that things might be otherwise than they are. I believe, and so do you, that things could have been different in countless ways. But what does this

mean? Ordinary language permits the paraphrase: there are many ways things could have been besides the way they actually are. On the face of it, this sentence is an existential quantification. It says there exist many entities of a certain description, to wit 'ways things could have been'. I believe things could have been different in countless ways; I believe permissible paraphrases of what I believe; taking the paraphrase at its face value, I therefore believe in the existence of entities that might be called 'ways things could have been'. I prefer to call them 'possible worlds'.³⁶

Here is another common strategy:

4. A is a paraphrase of B
5. A does not imply that Fs exist.
6. So, B does not imply that Fs exist.

It is described in the following passage from W.V.O. Quine's 'Logic and the Reification of Universals':

Another and more serious case in which a man frees himself from ontological commitments of his discourse is this: he shows how some particular use he makes of quantification, involving a *prima facie* commitment to certain objects, can be expanded into an idiom innocent of such commitments. In this event the seemingly presupposed objects may justly be said to have been explained away as convenient fictions, manners of speaking.³⁷

Taking paraphrase to be a symmetric relation (if B is a paraphrase of A then A is a paraphrase of B) that is truth preserving, the first premises of these arguments imply each other, as do A and B on the condition that those premises are true. If A and B are equivalent then both arguments are valid and the conclusions together are contradictory. So, (still assuming the equivalence of A and B) the second premises are also contradictories, and the resolution must come by determining which of (2) or (5) is false.

The support for (2) or (5) might come from a superficial reading of either A or B. If there is a need for one of these arguments at all, the commitments suggested by these superficial readings will not be the same. So, the focus turns to which of the readings is more *fundamental*. There

³⁶ David Lewis, *Counterfactuals* (Cambridge, Harvard University Press, 1973), 84.

³⁷ Willard Van Orman Quine, "Logic and the Reification of Universals," in *From a Logical Point of View* (Cambridge, Harvard University Press, 1980), 103,4.

are various reasons for taking one reading to be more fundamental. It might be argued that the single proposition expressed by both A and B has a logical structure and one or the other sentence best represents that structure. Alternately, it might be argued that one of the readings better suits our linguistic purposes than the other and so should be regarded as more fundamental. It might also be maintained that the reading that does the least damage to pre-philosophical common sense is the most fundamental. Without planting squarely on a firm reason why, I will take the approach that the reading with the *least troublesome implications* is the one best-suited to fill the role of fundamental reading.³⁸ I do not think we should count ontological implications among the relevant, troublesome implications, in large part because it would not be dialectically useful to defend premises in debates of ontology by appealing to a desired ontology.

Ontological Commitment, How to Avoid it, and When you Cannot

The determination of which reading is fundamental is not, on its own, decisive in favor of (2) or (5). One must also be able to discern the ontological commitments of the fundamental reading. Let us suppose that sentence P is ontologically committed to Fs if and only if the following argument is valid ((7) necessitates (8)):

7. P
8. So, Fs exist.

This characterization of ontological commitment has the unfortunate consequence that if Fs exist necessarily then every sentence is ontologically committed to them.³⁹ For example, understanding ontological commitment this way, if numbers exist necessarily then nominalism

³⁸ I say something about fundamentality since I mention it; however, I do not employ it in the arguments below in a way that requires being clear about just exactly what it is.

³⁹ See Frank Jackson's "A Puzzle About Ontological Commitment" in J. Heil (Ed.) *Cause Mind and Reality* (Dordrecht, Kluwer Academic Publishers, 1989), 191-199. Also see Michaelis Michael, "Implicit Ontological Commitment," *Philosophical Studies* 141, no.1, (2008): 51.

(about numbers) is ontologically committed to numbers. However troublesome this seems at first, it is not any more worrisome than holding that a contradiction entails the law of non-contradiction because necessary falsehoods entail everything.

The dominant view in ontology (at least since the publication of ‘On what there is’ in 1953) has been that if P necessitates something appropriately representable as $\exists x(Fx)$ where the quantifier is understood objectually, then P necessitates that Fs exist.⁴⁰ There are alternatives to that account. Some maintain that only *heavyweight* or *ontologically loaded* objectual, particular quantifiers necessitate existence claims. Others reject that existence is captured by use of any quantifier; it is better understood as a predicate that need not be coextensive with the domain of objects over which the quantifiers range.

So, suppose that a set of sentences entails “there are Fs” or “something is (identical to) c.” Call that sentence ‘S.’ There are at least four ways of denying that the set is ontologically committed to Fs or c: (A) Find an implication-free paraphrase of S and show that the troublesome implications are minimized by taking the paraphrase to better capture the fundamental form of S. (B) Argue that S is not best represented by an objectual quantifier. (C) Argue that though S is well-represented by an objectual quantifier, it is ambiguous and might not be best represented by a heavyweight quantifier. (D) Grant that S is represented by an objectual quantifier but reject that it necessitates that some F or that c has the property of existence.

I will argue of a narrowly circumscribed set of natural-language sentences that they are ontologically committing. If a sentence S meets the three conditions below then none of the

⁴⁰ For example, Donald Davidson assumes it in his influential “The Logical Form of Action Sentences” in *The Logic of Decision and Action* ed. Nicholas Rescher, (Pittsburgh, University of Pittsburgh Press, 1967), and David Lewis assumes it in the quote from *Counterfactuals* above. Other authors cannot even comprehend denying it, e.g., William Lycan in “The Trouble with Possible Worlds” in ed. Loux *The Possible and the Actual* (Ithaca, Cornell University Press, 1979), 290.

escapes (A)—(D) of the previous paragraph can be satisfactorily employed to dodge ontological commitments.

- I. S is a simple, existentially quantified claim.
- II. S does not have a more fundamental paraphrase.
- III. S is equivalent to a numerically quantified sentence.

Let a simple, existentially quantified claim (SEQ) be a sentence of the form ‘something is F,’ ‘something is c,’ ‘there is an F,’ ‘there are Fs,’ ‘There is something that is c,’ or some other sentence that at least appears to be well-represented by ‘ $\exists xFx$ ’ or ‘ $\exists x(x=c)$.’⁴¹ If S is an SEQ that also satisfies condition II then that apparent form is its fundamental form—or at least it is best understood to make use of the particular quantifier. Condition II rules out sentences like ‘there is a good chance of rain.’ There will be little resistance to the fundamentality of the quantifier-free paraphrase ‘It is probable that it will rain.’⁴² An SEQ that satisfies condition II (a ‘fundamental SEQ’ or ‘FSEQ’) will not avoid commitment to Fs or c by means of a more fundamental paraphrase.

A sentence of the form ‘something is F’ meets condition III if it necessitates and is necessitated by a sentence of the form ‘at least one thing is F.’ A sentence of the form ‘some things are F’ satisfies condition III if it co-necessitates ‘at least two things are F.’ A sentence of the form ‘something is c,’ if it met condition III, would be equivalent to ‘there is exactly one c.’ If a sentence S that satisfies condition III is an FSEQ then the equivalent numerically quantified

⁴¹ Here ‘c’ stands in for a singular term. I ignore the corner quote convention and allow substitution for variables within inverted commas. I do not think there will be occasion for confusion to result.

⁴² I have adapted an example of Frank Jackson’s from “Ontological Commitment and Paraphrase” *Philosophy* 55, no. 213 (1980): 306. His point in giving the example is *related* to that above; paraphrase is reflexive and so entailments of a sentence belong to its paraphrase. Consequently for Jackson, paraphrase cannot be used to escape commitments. Jackson might resist the claim that the paraphrase is more fundamental. So, perhaps I should say ‘excepting for the originator of the example, there will be little resistance.’

sentence will not be more fundamental than S; it will be on par or less fundamental. Condition III rules out sentences like ‘there is gold in those hills’ and ‘some waters are rough.’⁴³

Consider adding a condition IV:

IV. S is not derived from a set of sentences for which a more fundamental paraphrase is possible to a set from which no existential generalization is possible.

Condition IV is similar to condition II; in some cases an SEQ might not have an adequate paraphrase, though it is derived from sentences that do have more fundamental paraphrases.

Consider two arguments—the first adapted from Stephen Schiffer’s *Remnants of Meaning* and the second from Thomas Hofweber’s “A puzzle about ontology:”

9. Mother Theresa has the property of being humble.

10. So, there is a property of being humble.⁴⁴

11. Fred admires Sherlock Holmes.

12. So, something is Sherlock Holmes.⁴⁵

It is arguably true that there are no adequate, (commitment-and-) quantifier-free paraphrases of (10) and (12). However, as Schiffer points out, (9) is a ‘pleonastic’ paraphrase of ‘Mother Theresa is humble’ which does not have a structure that obviously permits the inference to (10). Though it is not Hofweber’s approach, one possibility for avoiding the commitment of (11) to Sherlock Holmes is to find a paraphrase that does not have the form: Admires<Fred, Holmes>.

⁴³ Kris McDaniel points out mass-quantification counterexamples of this type in “Ways of Being” in *Metametaphysics* ed. David Chalmers, (New York, Oxford, 2009), 300. He credits K. Koslicki with bringing them to his attention. However, it might be overly cautious to rule them out as I have since they could be rewritten as (the numerically paraphrasable) ‘there is a *quantity* of gold in those hills’ and ‘some *quantities* of water are rough.’ See Michael McKinsey, “Apriorism in the philosophy of language,” *Philosophical Studies* 52, no. 1 (1987): 13,4.

⁴⁴ Stephen Schiffer, *Remnants of Meaning* (Cambridge, MIT Press, 1987), 235-237.

⁴⁵ Thomas Hofweber, “A Puzzle about Ontology.” *Noûs* 39, no.2 (2005): 272. Both Schiffer and Hofweber use as a conclusion an SEQ that would be made true respectively by the property of humility and by Sherlock Holmes, e.g., ‘there is something Fred admires, namely Holmes.’ I have changed the conclusions to SEQs.

Both Schiffer and Hofweber take the premises to be true and the arguments to be valid. However they do not think the existence of humility and Sherlock Holmes should be so easy to establish. So they reject the commitment of the conclusion.

It is tempting to add condition IV to avoid the escape route by ruling it out *ad hoc*. However, if the argument below is sound then conditions I-III are sufficient for ontological commitment and adding condition IV would simply be a maneuver for avoiding a justified criticism. So I will consider a variant of the criticism below in the section titled ‘the triviality argument.’ My general response is this: a paraphrase of (9) or (11) (or any discourse) to avoid ontological commitments could only be accomplished by virtue of its no longer implying (10) or (12) (or a similar offending SEQ). If any suitable paraphrase of (11) implies (12) then (11) is ontologically committed to Sherlock Holmes in case (12) satisfies I-III. The situation is similar for (9) and (10). If (10) satisfies I-III then (9) is not trivial and must not be a paraphrase of ‘Mother Theresa is humble,’ if that sentence is trivial.

First I will argue that sentences that meet conditions I-III are ontologically committed. Then I will argue that SEQs derived by an application of EG from logical truths like $a = a$ (and the mathematical claims entailed by our adequate scientific theories) meet the four criteria.

Sentences Satisfying I-III are Best Represented by an Objectual Quantifier

So consider an FSEQ that satisfies III. It will clearly avoid escape (A); it will have no more fundamental paraphrase. It will also avoid escape (B); it will be best represented by an *objectual* quantifier. Here is why: the alternatives to the objectual quantifier are the substitutional quantifier and the inferential role quantifier, but condition III rules out both.

I will not consider either alternative in great detail. I will just give a rough account of each and then offer an argument that an FSEQ satisfying condition III is not well-represented by

either. A substitutional rule for the particular quantifier makes ‘ $\Sigma x Fx$ ’ true if some substitution instance of ‘ Fx ’ is true, where substitution instances are obtained by replacing ‘ x ’ with some element of a class of terms (called a substitution class).⁴⁶ Hofweber’s inferential role interpretation of the quantifiers makes ‘the quantified statement...truth conditionally equivalent to the [infinite] disjunction over all of the [natural language] instances that are supposed to imply it.’⁴⁷ This makes an inference like existential generalization ‘trivially valid’ and gives the quantified statement its inferential role. Hofweber notes the similarity between his inferential role quantifiers and substitutional quantification; I think that similarity allows condition III to rule out both.

It is noteworthy that some authors have taken the substitutional particular quantifier to be a “primitive device of infinite disjunction.”⁴⁸ Interpreting substitutional quantifiers as objectual quantifications over expression types seems to commit one to the existence of expression *types*. Consequently, those inclined to decrease ontological commitments to abstracta by appeal to substitutional quantifiers have reason to interpret them as infinitary disjunctions. So, on at least one account, what Hofweber calls ‘inferential role quantifiers’ are an interpretation of substitutional quantification, not an alternative to it. From now on I will talk just of substitutional quantification and mention the varieties of it if necessary.

Since there are two kinds of statements that satisfy conditions I through III (‘There are Fs’ and ‘something is c ’), I will argue for the cases separately.

Case I: There are Fs.

⁴⁶ It is conventional to use ‘ Σ ’ to represent a particular substitutional quantifier and ‘ Π ’ for a universal substitutional quantifier and reserve ‘ \exists ’ and ‘ \forall ’ for objectual quantifiers.

⁴⁷ Hofweber, “A Puzzle About Ontology,” 274.

⁴⁸ Hartry Field, “Review of *Ontological Economy: Substitutional Quantification and Mathematics*, by Dale Gottlieb,” *Noûs* 18, no.1 (1984):162.

Assuming condition III is met, ‘there are Fs’ is equivalent to either ‘at least one thing is F’ or ‘at least two things are F.’ The numerically quantified paraphrases are consistent with ‘the only Fs are unnamed.’ However, understanding the quantifier in ‘there are Fs’ as a substitutional quantifier (with a substitution class comprising names) would not preserve that consistency. For an example consider the FSEQ,

13. There are zedonks.

I take it that (13) is equivalent either to (14) or to (15), which one will not matter.

14. There is at least one zedonk.

15. There are at least two zedonks.

Both are consistent with

16. The only zedonks are unnamed.

The rule for a substitutional particular quantifier is roughly: ‘ ΣxFx ’ is true iff some substitution instance of ‘Fx’ is true, where a substitution instance of ‘Fx’ is obtained by replacing ‘x’ in ‘Fx’ with an element of the substitution class. If the substitution class is names in the language, then (16) is *inconsistent* with either (14) or (15). The appropriate representation of (16) is universally quantified; whether or not that quantifier is interpreted objectually or substitutionally the inconsistency remains. If (15) interpreted substitutionally is true then there is a true substitution instance of ‘x is a zedonk.’ If so then there is a named zedonk. That is clearly inconsistent with (16) interpreted objectually. If (16) is interpreted substitutionally then it can only be true vacuously. Denying this leads to no formal contradiction that I can see but it does lead to an undesirable absurdity; there would be some true sentence similar to ‘Bob is a zedonk and is unnamed.’ Of course, if (16) is true vacuously then (14) cannot be true.

One might dodge the undercounting problem by allowing the substitution class to include descriptions as well as names.⁴⁹ It is plausible that there is a natural language description available for everything we would have occasion to count (possibly this requires including indexicals and demonstratives). Of course, if there is one description (D, say) then there is another (for one: what is denoted by ‘D’). This leads, I think, to an insurmountable overcounting problem. To avoid overcounting, it is imperative to know when identity statements are true. For example,

17. There are at least two (current) U.S. Presidents.

If it is appropriate to represent this sentence with substitutional quantifiers then there seems to be no better representation than,

18. $\Sigma x \Sigma y [Px \cdot Py \cdot \sim (x = y)]$

The natural language sentence (17) avoids erroneously being true on the substitutional interpretation because, for one thing,

19. The Commander in Chief is (identical to) Barack Obama.

Identity statements with descriptions as in (19) can be represented either as quantified statements with ‘=’ taking small scope or as having ‘=’ take large scope over the relata. (e.g., ‘The one and only Commander in Chief is such that it = Barack Obama’, or ‘=<the Commander in Chief, Barack Obama>’)

If the description takes large scope over ‘=’ as it does on the standard Russellian account there are two options for the quantifier: it could be interpreted objectively or substitutionally.

20. There is an x that is a commander in chief and for all y if y is a commander in chief then y is identical to x, and x is identical to Barack Obama.

⁴⁹ Joseph Camp, for one, suggests allowing descriptions in the substitution class in “Truth and substitutional quantification,” *Noûs* 9, no. 2 (1975): 165-185.

Interpreting the particular quantifier objectually requires for the truth of (20) that there be a domain element that satisfies the description. This obviates the advantage of appealing to Σ to avoid ontological commitment since it carries the same commitment to domain elements as \exists . So instead, consider interpreting the leading particular quantifier substitutionally. The truth of (20) will depend in part on whether there is a true substitution instance of ‘x is identical to Barack Obama.’ The possibility of unnamed *property instantiators* led us to accept a substitution class containing descriptions. So identifying the truth conditions for (19) with those of (20) will take us no closer to evaluating the truth of (19). If ‘the commander in chief’ is the sole description in the substitution class then the truth of (19) will be indeterminate—it will depend on the truth of (19). If there is some other description D that might satisfy ‘x is identical to Barack Obama,’ then the question of whether it does will require knowing already that D is identical to Barack Obama or else knowing that some other description D' satisfies ‘x is identical to Barack Obama.’ This regress might be fortuitously avoided if some name could be truly substituted for x. However, this is not guaranteed given our reason for expanding the substitution class to include descriptions. This interpretation of (19) leaves its truth indeterminate.

The remaining alternative is to treat the identity in (19) as having large scope. We are considering Σ as a way to avoid commitment to domain elements so the standard truth rule for ‘a = b’ is unavailable:

21. It is true that a = b if and only if the denotation of ‘a’ is identical to the denotation of ‘b.’

In virtue of what then could the equality in (19) be true? Perhaps truth conditions could be given by a Leibnitz-like Law (with universal substitutional quantification represented by ‘ Π ’—I take it

this would be more appealing than second order objectual quantification to someone who uses to substitutional quantification to avoid commitment to domain elements):

22. It is true that $a = b$ if and only if IIS (if S is an open sentence (without identity) then S/a is true if and only if S/b is true)

Of course, $x = b$ is an open sentence; so to be helpful the rule has to exclude open sentences that contain identity. However, this will not work as an account of the truth conditions for $a = b$ if there is any hope of its being applied to non-extensional operators; it falters on opaque contexts—even on those that are not explicable as quotation contexts. Note that in each of the following pairs of sentences, the second (and false) sentence results from the first (and true) sentence by substitution of a co-denotative description.

- A. Barack Obama won the 2008 presidential election because he was *the recipient of the majority of electoral votes cast for president in 2008*.
- B. Barack Obama won the 2008 presidential election because he was *the winner of the 2012 presidential election*.
- A. Barack Obama was forty-seven years old when he became *the recipient of the majority of electoral votes cast for president in 2008*.
- B. Barack Obama was forty-seven years old when he became *the winner of the 2012 presidential election*.

Using (22) as a rule for identity would result in over-counting. It would be true that there are at least two current U.S. presidents (17), since by virtue of the substitution failures above, (23) is false.

23. The recipient of the majority of electoral votes cast for president in 2008 = the winner of the 2012 presidential election.

Saul Kripke takes opaque contexts (he considers necessity, propositional attitudes, and quotation) to be reason to “be careful about ‘identity’ in a substitutional language...In the absence of [a totally defined denotation function for all the terms of L] there is no obvious way to interpret ‘=.’” Kripke, in a footnote, considers defining identity in a way that depends on the identity of indiscernibles and cites a drawback similar to the one mentioned here.⁵⁰

The problem with taking ‘at least two things are Fs’ as being representable by Σ —at least as I have presented the problem—is this: such a representation relies on equality statements that either have mysterious truth conditions or have truth conditions that rely on domain elements (when the point was to avoid such reliance). There is at least one promising way to represent numerical claims like ‘there are at least two Fs’ without using ‘=.’ Generalized quantifiers allow ‘there are at least two’ to act as a quantifier itself. These are generally (universally, from what I can tell) taken to be objectual, but conceivably a substitutional account could be given. This would require, I believe, a partitioning of the terms in the substitution class. Such a partition into equivalence classes would have to overcome the same difficulties that the equality statements faced.

Case II: Something is c.

The straightforward substitutional interpretation of ‘something is c’ is

$$24. \Sigma x x = c$$

The problem with this interpretation is much the same as the problem for case I; there are no obvious, non-objectual, truth conditions for simple identity statements. (24) will be true just in

⁵⁰ Saul Kripke, “Substitutional Quantification,” in *Truth and Meaning: Essays in Semantics*, ed. Garreth Evans and John McDowell, (New York, Oxford, 1976), 350.

case some n in the substitution class makes it true that $n = c$. But in virtue of what would that be true? If the point of appeal to Σ is to eliminate appeal to domain elements then it cannot be that $n = c$ is true just in case the denotation of 'n' is the same as the denotation of 'c.' Allowing the truth of basic identity sentences to be determined by convention or by story threatens to permit violations of transitivity. R. B. Marcus defines a logical constant 'I' for use in representations like (24) by the principle of substitutivity.⁵¹ Her truth conditions for identity statements look very much like (22). She eliminates troubles based on quote names by limiting the substitution class of open sentences to those formed by replacing names and not quote names (or parts thereof) with placeholders. She also paraphrases 'John does not know that Cicero is Tully' as 'John does not know that Cicero is called "Tully"' in order to handle cognitive verb counterexamples by the same quote name restriction. The examples of substitution failures in the preceding section cannot be used again here. However, even supposing that some substitution rule could be used for identity statements, there is an immediate problem for a substitutional interpretation of 'something is identical to c.'

It should be clear from the preceding section that a substitutional interpretation of 'something is F' is inadequate. That is sufficient to show that a substitutional interpretation of 'something is c' is also inadequate. If the patchwork semantics that would result from denying this is not problem enough, there is another problem; it would lose the following, obviously valid argument:

Something is (identical to) c and c is F.

So, something is F.

Since the conclusion is only adequately represented by an objectual quantifier, the only adequate quantifier for the premise is also objectual. Someone trying to dodge ontological commitments

⁵¹ Ruth Barcan Marcus, "Quantification and Ontology," *Noûs* 6, no.3 (1972): 248.

by appeal to Σ should not be content with an objectual quantifier's being so entailed by a substitutional one.

I do not see how inferential role quantification fares any better with identity (in either case I or II) than the sort of substitutional quantification that I have considered; to avoid overcounting a truth rule for identity statements is required and it is difficult to come by an adequate rule without appeal to denotations.

Sentences Satisfying I-III are Univocal

Sentences satisfying conditions I-III are best represented by objectual quantification. I now argue that all such sentences are heavyweight. The proposal I am criticizing is that some objectually quantified sentences are heavyweight and others are lightweight; that is, some have ontological implications and others do not (or have implications of a lighter sort—e.g., a different mode of being). My response is that sentences satisfying the conditions are univocal and some of them are heavyweight, so all of them are. The argument originates with Peter van Inwagen. His argument is roughly this: Quantificational claims are existence claims, Existence claims are 'closely allied' with numerical claims. Numerical claims are univocal. So, quantificational claims are too.⁵² Since, for us, sentence S is equivalent *by supposition* to a numerically quantified sentence, my version of the argument is easier to defend. Consider again: 'something is c.' It is equivalent by supposition to 'at least one thing is (identical to) c.' Assuming the transitivity of identity; that is equivalent to,

25. Exactly one thing is c.

There is no reading of sentence (25) that is not paraphrasable as,

26. The number of things that are (identical to) c is one.

⁵² Van Inwagen, Peter, "Being, Existence, and Ontological Commitment," in *Metametaphysics* ed. David Chalmers (New York, Oxford, 2009), 480-483. Also, a similar argument appears in "Meta-Ontology" by Peter van Inwagen in *Erkenntnis* 48 (1998): 235-237.

That (26) is not the most fundamental paraphrase of ‘something is c’ is assured by its difference in surface structure and by condition II. That will not matter for the argument. Van Inwagen points to the ‘univocacy of number’ as evidence that (26) is unequivocal.

Since there are not heavy and light readings of ‘one,’ no reading of (25) that is equivalent to (26) cannot have heavy and light readings. In van Inwagen’s words,

The essence and applicability of arithmetic is that numbers can count anything, things of any kind, no matter what logical or ontological category they may fall into: if you have written thirteen epics and I own thirteen cats, the number of your epics *is* the number of my cats.⁵³

In response, Jason Turner has pointed out that an ambiguity in ‘one’ is not the only possibility for ambiguity. Someone committed to a heavy and light reading of ‘there are’ could as easily be committed to different readings of ‘the number of...is.’ As he puts it, “if we think there are different candidate ways for things to exist, there is nothing particularly embarrassing about thinking that there are different kinds of numbering relations as well.”⁵⁴ I think van Inwagen by ‘univocacy of number’ means to rule out multiple readings of both ‘one’ and of ‘the number of.’ Turner’s point remains however; it is not immediately obvious that ‘the number of’ is univocal.

Suppose that ‘the number of’ is equivocal. Then there is a true reading of,

27. The number of Fs is zero and the number of Fs is one.

There *is* an ambiguity between natural numbers and counting numbers so that zero is a number in one sense but not another. However, that does not provide the thick/thin distinction (nor does it provide a true reading of (27)).

⁵³ Van Inwagen, “Being, Existence, and Ontological Commitment,” 482.

⁵⁴ Turner, Jason, “Ontological Pluralism,” *Journal of Philosophy* 107, no. 1 (2010): 25,6.

Furthermore, in contexts ascribing quantities ‘is’ has an ‘is (at least)’ and an ‘is (exactly)’ reading. For example,

“Everyone in the party must be twenty-one for you to purchase that. Is she twenty one?”

“Yes. She is twenty-one. In fact, she is thirty!”

So it is conceivable that there be examples of the same ambiguity involving ‘the number of...is’. For example,

“I cannot see that far, does he have 270 electors yet?”

“Yes. The number of his electors is 270. The number of his electors is exactly 300.

Even if there is a non-contradictory reading of the response, it still does not help since it cannot be used to get the thick/thin distinction. It is difficult to find a thick/thin substitution instance of (27) that sounds remotely uncontradictory. (The number of Santas is zero and the number of Santas is one?)

If the question ‘What is the number of Fs?’ is paraphrasable as ‘How many Fs are there?’ as it seems to be, then the answer to the latter question must await a disambiguation of the question. It is difficult to imagine a context in which the question could be taken as ambiguous. Perhaps in metaphysics class: ‘how many objects are there in a world with two simples?’ Even in this case the *question* seems to be clear despite not having a clear answer.

One major problem for the ambiguity view comes from possibility of performing simple operations on numerical claims. Consider the following claims in the context of a discussion of Greek mythology and real textiles,

(L) The number of golden fleeces is one.

(H) The number of non-golden fleeces is ninety-nine.

Suppose, (L) and (H) are both true—the first only in the light sense because of the mythical tale of Jason and the Argonauts and the second because of a real, world-wide moth outbreak, say; it would seem to follow that,

(C) The total number of fleeces is one hundred.

This would not follow if (L) and (H) contained different senses of ‘number of’ (as they would have to in order to both be true in a single context), since (C) would be true for neither of the two senses of ‘number of.’⁵⁵ It is odd to add the number of fictional fleeces to the number of real ones, but that oddity is not a result of a category mistake as it would be trying to apply the addition function to arguments in different categories. Compare to the absurdity of

(S) The shape of Jörmungandr⁵⁶ is isomorphic to the shape of your tennis racket.

The expressed relationship is fine unless ‘the shape of’ is used in two different senses. If the first occurrence of ‘the shape of’ is used in the topological sense and the second is used to mean ‘condition’ as it is in answering the question ‘what shape is your racket in?’ with ‘the shape of my racket is good,’ then (S) is a category mistake.

A similar argument can be made with subtraction. For his play, *The Crucible*, Arthur Miller amalgamated into one judge (Danforth), several judges who presided over the Salem witch trials. Suppose that the actual number of presiding judges was three.

(D) The number of presiding judges differs (in the story and in reality) by two.

(D) intuitively follows from the facts above, but it would not be true in either proposed sense of ‘the number of.’ It might be argued that the subtraction is impossible because the fictional judge and the real judges have a different mode of being, but this is a different objection than saying

⁵⁵ This is unlike the addition of natural numbers and real numbers. That is possible, but only because the naturals are a subset of the reals. So the sum is a real number. A similar subset approach to heavy/light numbers is nonsensical.

⁵⁶ A creature of Norse mythology, Jörmungandr is a serpent wrapped around the earth, biting its own tail.

‘the number of’ is ambiguous. (It is an objection that fails if the argument in the next section is sound.)

If there were heavy and light readings of ‘the number of’ then the heavy one might be differentiated from the light one by means of emphasis. For example, ‘the number of unicorns is too high to count but the *real (or true)* number of unicorns is zero.’ This is no less contradictory than it is without the ‘real (or true).’ The same point about the adjective ‘real’ serves as a response to another objection to the van Inwagen argument.

Sentences Satisfying I-III are Heavyweight

Someone might grant the univocacy of ‘there is’ but insist on its being unequivocally *lightweight*. That is, one might grant that ‘there are Fs’ is best represented by an objectual quantifier but deny that such a quantifier implies the existence of Fs. However, just as ‘the number of Fs is one’ is equivalent to the ‘the *real or true* number of Fs is one’⁵⁷ so ‘there is one F’ is equivalent to ‘there *really* is one F.’ The latter sentence of each pair is plainly heavyweight, so given univocacy, ‘there is’ is heavyweight too.

This simple argument, if sound, goes a long way. I will consider objections to the argument and objections to the thesis for which it is an argument. ‘There really is an F’ implies that an F exists; consequently, so does the paraphrase ‘there is an F.’ This is a paraphrase argument of the first type mentioned at the outset of the chapter. So, there seem to be two possible criticisms: deny the ontological commitment of ‘there *really* is an F,’ or deny the paraphrase. Accepting the paraphrase but denying the commitment of ‘there really is an F’ is possible. I have no real argument against this (prima facie absurd) position, but consider a natural response to someone denying the existence of witches: ‘there *really are* witches.’ If the

⁵⁷ It is contradictory to say that the number of Fs is one but that the real or true number of Fs is not. (Compare: there is one F but not really.)

existence of witches is not implied by that response, it is implicated; and it is tough to see how that would be so (flouting Relevance?).

Jody Azzouni, for one, denies the ontological commitment of ‘there really is an F.’ However, he also rejects the commitment of ‘Fs exist.’⁵⁸ I think he would grant that ‘there really is an F’ implies that ‘Fs exist.’ So, we (merely) disagree over what ontological commitment is.

Consider then a denial of the paraphrase: as a practical matter, why should anyone add the word ‘really’ to a quantificational claim if it has no effect on truth value? I think this pragmatic objection can be simply addressed. One might add ‘really’ to a quantificational claim for the same reason that she typically adds ‘really’ to any sentence: to show that she is not speaking non-literally—that she is not using overstatement or being metaphorical or sarcastic. Invariably, if ‘Really, X’ is true then ‘X’ is true. The same point can be explained in different terms: there is a difference between claims that are acceptable in a context and claims that are true. The insertion of ‘really’ suggests that the claim uttered is to be taken as true rather than as merely acceptable.

One possible way of at once denying both the ontological commitment of ‘there really are Fs’ and the paraphrase is to say that ‘really’ acts as an actual world operator, so that ‘there are Fs’ and ‘there really are Fs’ express different propositions (the contingent truth of the former in the actual world would imply the necessary truth of the latter). If so, then ‘there really are Fs’ would not necessitate ‘Fs exist’ because there would be worlds where there are no Fs despite its being true in those worlds that there are Fs in the actual world. If validity is understood as real world validity (there are good reasons to think it should not be) then this objection does not get off the ground because there is no model (domain and denotation) where ‘there are Fs’ is true

⁵⁸ Azzouni, Jody, “Ontological Commitment in the Vernacular,” *Noûs* 41. No.2 (2007): 209.

and ‘Fs exist’ is false. On the other hand, if validity is defined in terms of worlds, then the commitment fails.

Perhaps ‘there really is an F’ is ambiguous between the redundant use of ‘really’ and a modal use equivalent to an actuality operator, but this does not undermine the simple argument of this section. As long as (1) there is a redundant use of ‘really’ that is ontologically committed (there obviously is) and (2) every use of the univocal ‘there are Fs’ is equivalent to the redundant interpretation of ‘there really are Fs’ (it seems to be) then my argument goes through. Even if it were plausible that ‘actually, there are Fs’ does not necessitate ‘actually, Fs exist,’ it is irrelevant that the former is non-committal since it is not equivalent to ‘there are Fs’. Compare: ‘Bob held a flying nocturnal mammal.’ Since that is equivalent to ‘Bob held a bat’ it implies that ‘Bob held a rodent.’ It does not undermine that conclusion to note that ‘Bob held a bat’ could also be interpreted to mean ‘Bob held a stick used for striking a ball in a game.’

Now I turn to objections to the van Inwagen argument presented in this chapter. In ‘The Question of Ontology’, Kit Fine gives at least two arguments that quantificational claims (QC) are not ontological claims (OC).⁵⁹

Objection One: The Triviality Argument

Fine puts the argument (versions of which were introduced above by Schiffer and Hofweber) against “the commonly accepted view...that ontological questions are quantificational questions” this way:

[W]hatever the answer to the ontological question of whether numbers exist, it is neither trivially true nor trivially false; and similarly for the existence of chairs and tables and the like. However, the answer to the corresponding quantificational questions *are* [sic] trivial. Thus given the evident fact that there is a prime number greater than 2, it trivially follows that there is a number (an x such that x is a number); and similarly, given the

⁵⁹ Fine, “The Question of Ontology,” 158.

evident fact that I am sitting on a chair, it trivially follows that there is a chair (an x such that x is a chair).⁶⁰

A simple reconstruction of Fine's argument:

QC are trivial.

OC are not trivial.

So, QC are not OC.

In response one could conceivably maintain the ontological commitment of QC and accept the conclusion. However, what is really at issue is whether QC imply OC. Denying the converse seems to leave the real problem untouched: Even if OC do not imply QC, QC are trivial and if they implied OC then OC would be trivial too, but they are not. We can take Fine's conclusion as: QC do not imply OC, instead of as expressing inequality. It is tempting to deny what the implication-version of the argument seems to rely on—the closure of triviality under implication—since some implications are not obvious (and triviality depends for Fine on being *evident*). It is plausible that the implication from QC to OC is just such a non-trivial implication. However, even supposing that the implications from QC to appropriate OC are trivial, there is a bigger problem.

For Fine, the triviality of QC results from their following trivially from evident facts. Consider his example: 'There is a number' follows trivially from the evident fact that there is a prime number greater than two. On the other hand, Fine maintains that 'a number exists' is not trivial. However, that follows trivially from,

28. A prime number greater than two exists.

So, Fine is relying on (28) not being an evident fact. Of course, no one in need of convincing that OC do not imply QC will accept that (28) is less evidently true than 'there is a prime number

⁶⁰ Ibid.

greater than two.’ The argument thus begs the question. Notice that it is not a promising option to block the objection by appealing to the non-triviality of the implication to (28) from the corresponding QC, since the same claim of non-triviality would undermine Fine’s argument too. Though the argument lacks persuasive force, it remains for Fine’s opponent to say whether OC are trivial or QC are non-trivial. G. E. Moore would take the first option. I take the second. It is trivial and non-philosophical to determine whether some QC (There are prime numbers, There are chairs and tables) are *acceptable* (whether they follow from other acceptable claims like axioms or sensory reports). It is less trivial and more philosophical to determine whether those QC are *true*.

Objection Two: From the Relative Strengths of Realist Claims.

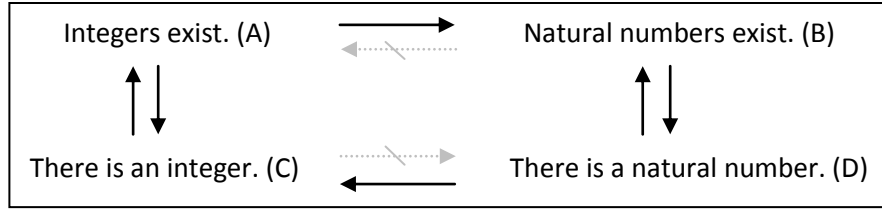
Here is how Fine puts the argument:

Consider a realist about integers; he is ontologically committed to the integers and is able to express his commitment in a familiar fashion with the words ‘integers exists’. [sic] Contrast him now with a realist about natural numbers, who is ontologically committed to the natural numbers and is likewise able to express his commitment in the words ‘natural numbers exist’. Now, intuitively, the realist about integers holds the stronger position...The realist about integers—at least on the most natural construal of his position—has a *thorough-going* commitment to the whole domain of integers, while the natural number realist only has a partial commitment to the domain.

However, on the quantificational construal of these claims, it is the realist about integers who holds the weaker position. For the realist about integers is merely claiming that there is at least one integer (which may or may not be a natural number) whereas the realist about natural numbers is claiming there is at least one natural number, i.e. an integer that is also nonnegative.⁶¹

I assume that Fine understands strength of position in the usual way: A is stronger than B if A implies B but not vice versa. Understanding the arrow to represent implication we could represent Fine’s argument as a *reductio ad absurdum*. Suppose that OC co-imply QC. Then there are two contradictions, evident in the following diagram.

⁶¹ Ibid. 166.



B, through D and C, implies A, but B does not imply A. Likewise, C, through A and B, implies D, but C does not imply D. So, OC do not co-imply QC.

It is immediately obvious that the A and B sentences are ambiguous. There are at least for possibilities for ‘Integers exist.’ (Each possibility is followed by a quantifier-free categorical claim that is most naturally interpreted the same way.)

- (1) Some integers exist. (e.g., Men are nannies.)
- (2) Typically, integers exist. (e.g., Tigers are fierce.)
- (3) All integers exist. (e.g., Dogs are mammals.)
- (4) The group (taken all together) of integers exists. (e.g., Bacteria are ubiquitous.)

There is only one reading that Fine could have in mind. Consider first the fourth reading of the A and B sentences, call them 4A and 4B. 4A does not imply 4B, since a property of a group need not be had by the proper parts of the group (fallacy of division).⁶² So, on that reading 4A would not be stronger than 4B. Similarly, the first reading is not what Fine has in mind since 1A is clearly not stronger than 1B (since 1B Implies 1A). The second reading cannot be what Fine has in mind since there do not seem to be any integers that fail to typify the integers (maybe 0? maybe the primes?). Even supposing that (2) is sensible, it surely would not be taken (by anyone) to be implied by C. The only remaining possibility is the third—it is the reading that expresses a ‘thorough-going commitment to the whole domain of integers.’ Obviously, the

⁶² Even if the reader disagrees on this point, it is obvious that on such a reading the C and D sentences do not imply (respectively) the A and B sentences. Consequently, contradiction is avoided.

quantificational construal of 3A would not be C. Likewise, the quantificational construal of 3B would not be D. The quantificational construals of 1A and 1B would be C and D respectively, but as we have seen, 1A does not imply 1B. So much for the reductio argument; no reading of A and B generates a contradiction.

A challenge remains: to explain the intuition that A is stronger than B. The obvious quantificational construal of 3A would be ‘every integer is something,’ or less succinctly, ‘of everything if it is an integer then there is something identical to it.’ Representing that in the standard way with the material conditional and objectual quantifiers gives:

$$29. \forall x (Ix \rightarrow \exists y (x = y)).$$

Like (29), the corresponding construal of 3B is true on every interpretation (except possibly the empty domain); so on the quantificational construal, A would not be stronger than B as it intuitively seems to be. To capture the intuitive asymmetry Fine recommends an existence predicate that is not co-extensive with the domain.

This is not directly a challenge to my thesis about simple existentially quantified claims, but it is related. If, as I argue, the existence of *c* can be expressed with a suitable sentence of the form ‘something is *c*,’ then it seems like the existence of a group of objects ought to be expressible by means of the existential quantifier too. So then, how to explain the apparent relative strength of 3A over 3B despite the triviality of (29) and its analogue for 3B?

I think (29) is an adequate representation of 3A and that the strength of A over B is merely illusory. Generally, to say that ‘All P are E’ is stronger than ‘All Q are E’ implies that it is possible that there is a P that is not a Q. (It also implies that it is impossible that there is a Q that is not a P.) The relationship between sentences C and D, specifically the fact that C does not imply D, shows that this (necessary) condition is met. It might be that with A and B we

mistakenly treat the necessary condition as a sufficient one, and thereby take A to be stronger than B. More simply put, when Q is a proper subset of P, sentences like ‘all P are E’ intuitively seem stronger than ‘all Q are E.’ Saying so is not attributing a widespread reasoning error, because typically the former sentences *are* stronger than the latter. However, there is no difference in strength when both sentences are necessarily true. Consider

30. (All) Visions appear.

31. (All) Religious visions appear.

These two sentences bear more than a superficial similarity to A and B. For one thing, it is not immediately obvious whether they are best represented as attributing a property to visions or as having quantificational structure in the consequent (e.g., every vision is such that there is someone it appears to). They also are similar to A and B in that they are necessarily true. My opponent will disagree on that point, but should agree on this one: intuitively (30) seems stronger than (31), but that is not much evidence that the two are not necessarily true. Once we have reason for thinking that both are necessarily (in this case, analytically) true, the intuition of a difference in strength carries little weight. Religious visions are a proper subset of visions and that makes (30) seem stronger than (31). So, if the A and B were both logically true the misleading intuition of a difference in strength can be explained by the relationship between the antecedents.

It might be that I am wrong and that the obvious quantificational construal of 3A is inadequate. There does seem to be a sense of ‘the integers exist’ that *expresses a thorough-going commitment* to the integers and clearly (29) does not do that since it could be true on an interpretation without any integers in the domain.

The quantificational reading of ‘the natural numbers exist’ that *expresses a thorough-going commitment* to the natural numbers but does not imply ‘the integers exist’ is:

$$\star \quad \exists x(x = 0 \cdot Nx) \cdot \forall x(Nx \rightarrow \exists y(y = x + 1 \cdot Ny))$$

The logical structure of this claim is admittedly far from the surface grammar, but this seeming drawback is quickly seen to be unimportant once one notices that the statement is really just the recursive definition of ‘natural numbers’ with appropriately placed existential quantifiers to express existence. The natural numbers are (defined recursively as) zero and the successors of natural numbers. So to say that all the natural numbers exist is to say that zero exists and that all natural numbers have an existing successor. The displayed sentence \star does not validly imply the reading of ‘the integers exist’ that expresses through-going commitment:

$$\star\star \quad \exists x(x = 0 \cdot Ix) \cdot \forall x(Ix \rightarrow [\exists y(y = x + 1 \cdot Iy) \cdot \exists z(x = z + 1 \cdot Iz)])$$

Since the integers are zero and all the successors and predecessors of integers, to say ‘the integers exist’ is to say that zero exists and that all integers have existing successors and predecessors. This reading of the ontological claims avoids Fine’s reductio argument since \star is not implied by ‘there is a natural number’ and similarly $\star\star$ is not implied by ‘there is an integer’. As desired, \star does not imply $\star\star$. Furthermore, with the additional premise that $\forall x(Nx \rightarrow Ix)$, $\star\star$ does validly imply \star .

Fine believes that this response is unacceptably ad hoc. He believes that there ought to be a uniformity to ontological claims, and consequently that there ought to be a general scheme for expressing commitment to Fs (chairs, real numbers, etc.,) that requires only plugging in ‘F’ to the schema.⁶³ I agree that there is a general scheme for ‘Fs exist;’ the quantificational account endorsed in this chapter provides the scheme for sentences of the form ‘some Fs exist’—namely

⁶³ Ibid.

‘there is an F.’ However, there is no reason to think that Fine’s schema requirement holds for sentences of the form ‘all Fs exist.’ Some of them can sensibly be understood as contingent and some cannot. The trivial ones (e.g., all the chairs exist) all have the same form (the form of 29), however, any that are possibly substantive seem to be a conjunction of existential generalizations. As an illustrative analogy, if there is a non-trivial interpretation of ‘the events of 7/20/1969 occurred’ then it must be equivalent to ‘the lunar landing occurred and the lunar flag-planting occurred and ...’”

Objection Three: Ambiguous Domain

In conversation, Larry Powers presented the following objection to the argument of this chapter:

There is no absolutely universal domain.

So, every quantified claim is ambiguous as to domain.

So, the existential quantifier is not univocally heavyweight.

Premise one can be justified by appeal to set-theoretic paradoxes related to the universal set. Since some sets contain themselves and others do not, we could quantify over those that do not. So a set of everything would contain them (those sets that do not contain themselves) as a subset. As is well-known, contradiction arises from supposing the existence of such a set—it cannot contain itself, nor can it not. So much for the possibility of quantifying over the elements of the universal set. It will not do in response to suggest simply that: there is no set of all sets that do not contain themselves (on pain of contradiction) so such a set is not an element in the universal domain.

There are two choices available to believers in unrestricted quantification. One is to reject the commonly accepted axioms of set theory that avoid the paradox by limiting set size

(and as a consequence, eliminating a universal set). This is Quine's approach. His set theory has come to be known as New Foundations set theory.⁶⁴ The other is to abandon the view that the members of the universal domain are the members of a set-like object. The universal domain is not a thing. This is Richard Cartwright's approach.⁶⁵

Timothy Williamson gives a revenge problem for both approaches. Whether sets or no sets make their way into interpretations, interpretations will still make their way into the domain (because we quantify over them to make claims of validity etc.). And it follows that the universal domain is subject to a Russell-style paradox. A predicate R can be defined: for (unrestricted) all x, x has R iff x is not an interpretation under which P applies to x. But there must (because R is a contentful predicate) be some interpretation under which for (unrestricted) all x, P applies to x iff x has R. Call one such interpretation I(R). Since I(R) itself must be in the domain of unrestricted universal quantification, 'x' can take it as a value. So it follows that, P applies to I(R) iff I(R) is not an interpretation under which P applies to I(R).⁶⁶

I make two minor points in response to Powers' objection, neither of which does full justice to the criticism. Powers' objection be taken two ways: (1) as an argument for a heavy and light quantifier and (2) as a criticism of my argument that the FSEQs are unequivocally heavy. It fails as (1) since the purported domain ambiguity does not provide the heavy/light distinction. Supposing the critic's point is correct that we cannot quantify over all that there is (you know what the critic means even if she cannot successfully say what she means), it follows only that there is always something outside of the domain of quantification; it certainly does not follow

⁶⁴ Willard Van Orman Quine, *From a Logical Point of View* (Cambridge, MA: Harvard University Press, 1980), 80-101.

⁶⁵ Richard Cartwright, "Speaking of everything," *Noûs* 28, no. 1 (1994): 1-20.

⁶⁶ Timothy Williamson, "Everything," *Philosophical Perspectives* 17, *Language and Philosophical Linguistics* (2003): 425.

that there are non-existent elements inside the domain of quantification. It fares better as (2) a criticism of my claim that quantifiers are unequivocally heavy.

However, to be a criticism, it would seemingly have to be a criticism of the premise in my argument that *numerical* claims are unambiguous. If every quantified claim were ambiguous as to domain then the FSEQ-equivalent numerical claims would be similarly ambiguous. So for example there would be non-contradictory readings of ‘the number of stars is seventy billion trillion and the numbers of stars is not seventy billion trillion.’ Because, of course, there would be no single true statement of the form ‘the number of stars is exactly __,’ since many conceivable domains provide many different numbers of stars. In fact, the statement ‘the number of stars is exactly x,’ would be more than $2^{70,000,000,000,000,000,000,000}$ ways ambiguous, given the possible combinations of stars alone. This is beyond absurd, but it does not tell us what exactly went wrong with Powers’ argument—just that something seems to have.

The view that universal quantification is possible is appealing to me, but the technical sophistication required of a semantics to escape Williamson’s revenge paradox is beyond the scope of this document (and the ability of its author). Williamson himself offers a semantic account that avoids it, as does Tom McKay.⁶⁷ In response, I simply maintain that the purported domain ambiguity would show only that an FSEQ could be equivocal with respect to (a multitude of) heavy interpretations, not that it could be heavy/light equivocal.

Two Kinds of Sentence that Satisfy I-III

If the forgoing argument has been sufficient to establish the ontological commitment of a certain class of sentences then it becomes a matter of concern which sentences satisfy the

⁶⁷ Thomas McKay, *Plural Predication* (New York: Oxford University Press, 2006), 149-151.

conditions. I will make the case that two classes of sentences satisfy conditions I-III and consequently carry ontological commitment. The first group is derived from mathematical explanations of physical phenomena and the second is derived from trivial identity sentences.

Type One: Mathematical Claims Derived from Scientific Explanations

Consider Alan Baker's explanation of cicada periods: Life-cycle periods that minimize intersection are evolutionarily advantageous. Also, prime periods minimize intersection. So organisms with periodic life-cycles are likely to evolve prime periods.⁶⁸ It follows that there are prime numbers, from which it is a consequence that,

32. There are numbers.

Since (32) seems to be well-represented by ' $\exists x(Nx)$ ' criterion I is satisfied. There does not seem to be a more fundamental paraphrase of (32), so condition II is met. Furthermore, there is no reading of (32) that does not co-necessitate

33. The number of numbers is at least one.

I will not defend the equivalence here except to encourage reflection on the seeming absurdity of a counterexample to the equivalence. Since the conditions are met, if the argument above is sound then (32) is committed to the existence of numbers.

However, that is not the same as to say that Baker's explanation is committed to the existence of numbers. It is possible that the explanation be given in a paraphrased form that is more fundamental than Baker's version and in a form that does not imply (32).⁶⁹ See the discussion above on potential condition IV.

⁶⁸ Alan Baker, "Mathematical Explanation in Science," *British Journal for the Philosophy of Science* 60 (2009): 614.

⁶⁹ For example, Juha Saatsi tries this in "The Enhanced Indispensability Argument: Representational versus Explanatory Role of Mathematics in Science," *British Journal for Philosophy of Science* 62 (2011): 149.

If the commitment of (32) does not guarantee the ontological commitment of a theory that implies it, why consider Baker's Cicada explanation at all? Why does it not follow that any utterance of (32) is committed to the existence of numbers? Because not all utterances of (32) meet conditions II or III. For example 'she did a number on him' implies 'there are numbers.' However, the utterance of (32) so derived could be paraphrased away and it has a reading that is not equivalent to (33)—since numbers done on people might not be countable. That (32) follows from Baker's explanation shows it meets the criteria for commitment. The only remaining question for the commitments of the explanation itself, is whether there is a suitable paraphrase for the explanation that does not imply (26); I take it this is the question of whether prime numbers are explanatorily indispensable.

Type Two: Existential Generalization from Trivial Identities

Consider a trivial identity sentence:

34. Bas van Fraassen is (identical to) Bas van Fraassen.

By the application of an inference like Existential Generalization it follows that,

35. There is something that is (identical) to Bas van Fraassen.

Since it seems to be well represented by a sentence of the form $\exists x(x=v)$, condition I is satisfied.

On condition II: there are some paraphrases that seem to be more natural, but none of those suggest a different logical structure (e.g., BvF is something) except for perhaps 'Bas van Fraassen exists' which is not a paraphrase someone can employ to avoid ontological commitment (at least not as I have defined it: S is committed to Fs if S implies that Fs exist).

So the question remaining is whether condition III is satisfied. (35) implies and is implied by,

36. The number of things identical to Bas van Fraassen is (at least) one.

Do not worry that (36) implies that there is a number and can be easily paraphrased to the seemingly more fundamental (35); even though it may be less fundamental than (35), if (36) is univocal then so must be (35) *provided that there is no reading of it that is not equivalent to (36)*. If there were only one way to take sentence A and if every way of taking sentence B were equivalent to sentence A then there would be only one way of taking sentence B (or maybe it is better to say that any ways of taking B would be equivalent to each other). Suppose this were not right, and B_1 was not equivalent to B_2 . By supposition each is equivalent to A, by transitivity of equivalence, B_1 is equivalent to B_2 .

So is there a reading of (35) that cannot be paraphrased as (36)? No. It would have to be a reading where something was $B \vee F$ but fewer than one thing was, or an interpretation where the number of things that were $B \vee F$ was one but nothing was $B \vee F$. The fact that $B \vee F$ is countable rules out the first possibility, and the fact that one $B \vee F$ is some $B \vee F$ rules out the second possibility.

Since there is nothing special about Bas van Fraassen (at least nothing that would prevent universal generalization), any existentially quantified sentence obtained from a trivial identity statement by an application of Existential Generalization satisfies conditions I-III and so entails the corresponding existence claim.

So, if existential generalization from simple existence claims is valid then it follows from the argument in this chapter that trivial identity claims entail existence claims. However, nothing in this chapter guarantees the validity of existential generalization on simple existence claims. In Chapter Two, I will consider the leading ways of rejecting existential generalization: bivalent positive free logic and supervaluational semantics.

CHAPTER TWO: POSITIVE FREE LOGIC AND SUPERVALUATIONAL SEMANTICS

Since a priori knowability is closed under logical implication, we will have to examine modification to the classical account of logical implication in order to avoid the absurd conclusion of the argument in the introduction. First, I will consider the two leading candidates for rejecting existential generalization. In this chapter I will explain and criticize bivalent positive free semantics and do the same for non-bivalent, supervaluational semantics.

Bivalent Positive Free Logic and the Prototype Schema

By jettisoning Existential Generalization one could escape the absurd conclusion of the (master) argument from a priori knowledge (1-5 in the introduction).⁷⁰ Consider,

1. Sherlock Holmes is a detective.

and,

2. It is false that the Greeks worshipped Father Christmas.⁷¹

Both appear to be true, but both resist existential generalization. Positive free semantics, which is largely the invention of Karel Lambert, accommodates the truth of 1 and 2 without requiring the existence of Sherlock Holmes or Father Christmas. One way of accomplishing this divides the domain into two parts: an inner domain over which the variables range and an outer domain composed of fictional or non-existent elements.⁷² Since Sherlock is in the outer domain, it could be true that Sherlock is a detective without it being true that there exists a detective that is identical with Sherlock. To avoid referring to non-existent objects, one might appeal to story

⁷⁰ It would also allow escape from the absurd conclusion of Burge's argument from non-referring singular terms. For more on that argument, see chapter three, section three.

⁷¹ This is adapted from an example in Priest, *An Introduction to Non-Classical Logic*, 130.

⁷² Lehmann, "More free logic," 221.

semantics according to which the interpretation contains a story function that assigns truth values to some sentences with non-referring terms.⁷³

It is standard in PFL that quantifiers range over the inner domain, so that sentences like ‘Sherlock Holmes exists’ (represented ‘ $\exists x(x = \textit{Sherlock Holmes})$ ’) does not come out true. Also, when quantifiers range over only the inner domain the truth of such contingencies as ‘all humans are mortal’ is maintained. This limitation on the quantifiers is the reason for PFL’s abandonment of EG. It is also responsible for the loss of more obviously valid inferences.

If a semantics for first order logic is an account of truth *and* entailment then the appropriate semantics for natural language should not contradict our clearest intuitions about the simplest inferences. Countless authors have illustrated validity with an argument for Socrates’ mortality. It can be validly inferred from the fact of his manhood and the mortality of all men. On a positive free semantics, that prototypical, valid argument is not valid. Consider an argument with the same form:

3. All horses are flightless.
4. Pegasus is a horse.
5. Therefore, Pegasus is flightless.

The universal quantifier in 3 ranges over only existing horses. Whether the truth of 4 derives from a member of the outer domain having the property of horse-hood or from a story that assigns it truth directly, 5 need not be true. What is true of the existing horses, according to either version, need not be true of horses in the outer domain or elements of a story. The loss of this valid form is too large a loss to tolerate. For the positive free logician, the argument could be made valid by adding ‘Pegasus exists’ as a premise. However, for such a premise to be required, many logicians must have been mistaken about the argument form for millennia.

⁷³ Ibid., 222.

An Appeal to Authority (in Defense of the Prototype Schema)

Call an instance of the following schema a ‘prototype argument’ since many authors use it as a first and most obvious example of a valid argument.⁷⁴

1. All A are B
2. x is A
3. So, x is B

The argument form appears frequently in twentieth century logic primers, because, I think, it is exemplary of *an obviously valid argument* that cannot be shown so by either the methods of classical syllogistic logic or of sentential logic. Indeed, as you would suspect, a version of it appears at the start of the chapter on quantificational logic in Copi’s *Introduction to Logic*.⁷⁵ However modern this reason is for emphasizing prototype arguments, thinkers have appreciated their validity for millennia. Here are a few examples selected to represent major periods in the history of thought.

For the modern period, in *The Port-Royal Logic* (1662), Arnauld and Nicole are satisfied to have shown an argument valid by reducing it to:

[All] Kings ought to be honored.

Louis XIV is king.

Therefore, Louis XIV ought to be honored.⁷⁶

Exemplary of the medieval period, in *Summa Logicae* (c. 1323), William of Ockham claims that this argument is valid:

⁷⁴ A reader of this document is likely to have encountered examples numerous enough to obviate my giving any. Despite this fact, I offer one productive example. Ian Hacking gives *dozens* of different prototype arguments in order to illustrate validity in *A Concise Introduction to Logic* (New York: Random House, 1972), 13-27.

⁷⁵ Irving M. Copi, *Introduction to Logic*. 1st ed. (New York, Macmillan, 1953), 278.

⁷⁶ Antoine Arnauld, and Pierre Nicol, *Logic; or, The Art of Thinking: Being the Port Royal Logic*, transl. Thomas Spencer Baynes, (Edinburgh: Sutherland and Knox, 1850); 209. It is notable that Arnauld takes ‘Louis XIV’ to be “standing for [a] universal” so the argument is a valid example in the first figure; so he must take ‘Louis XIV’ to have existential import.

Every man is white.

Socrates is a man.

Therefore, Socrates is white.⁷⁷

Exemplary of the ancient period, in *Prior Analytics* (c. 330 B.C.E.) Aristotle claims the following argument is ‘irrefutable if true:’

[All] Ambitious men are generous.

Pittacus is ambitious.

Therefore, Pittacus is generous.⁷⁸

He seems to mean that if the premises are true then conclusion is irrefutable; he claims that the argument is in the first figure. According to Bertrand Russell (in history of Philosophy p218), Aristotle does not distinguish the argument from Barbara (All A are B, All B are C so All A are C).

These examples are all from passages on logic. The point of the author, in each case, is not to establish a conclusion, but to *illustrate or give an example* of a valid argument. Expanding our samples to include arguments used rather than those mentioned,⁷⁹ the argument form is plausibly as old as philosophy itself.

If Herbert Granger is right then the surviving fragments of Heraclitus reveal to us several of his arguments, including fragment 85; ‘it is hard to fight with anger; for whatever it should want it buys from the soul.’⁸⁰ It is plausible that Heraclitus was relying on an unstated universal premise. If so, his argument would be:

⁷⁷ William of Ockham, *Philosophical Writings; A Selection*, transl. Philotheus Boehner (New York: Bobbs-Merrill, 1964), 94.

⁷⁸ Aristotle, *Prior Analytics*, transl. A.J. Jenkinson, in *The Complete Work of Aristotle, The Revised Oxford Translation*, ed. Jonathan Barnes (Princeton, Princeton University Press, 1991), Bk.II, Ch.27, 70a 26-30.

⁷⁹ Examples of the latter type contribute less to my appeal to authority than do the examples above since it is possible that authors use arguments that they do not take to be valid.

⁸⁰ Herbert Granger, “Argumentation and Heraclitus’ book,” *Oxford Studies in Ancient Philosophy* 26 (2004): 12.

[Everything that buys whatever it wants from the soul is hard to fight.]

Anger buys whatever it wants from the soul.

Therefore, anger is hard to fight.

So, if Heraclitus' argument is really enthymematic (in the contemporary sense not merely in the Aristotelian sense) then the prototype argument has been in use since the sixth century BCE.⁸¹

Not every author has found the prototype argument convincing. Sextus Empiricus, a skeptic, gave two criticisms of this argument:

Everything human is an animal.

But Socrates is Human.

Therefore, Socrates is an animal.⁸²

The criticisms that Sextus gives are widely applicable. He applies the criticisms to what he calls the "Stoic's Unprovables" which include modes ponens, modes tollens, and disjunctive syllogism. Neither of his criticisms claims that the arguments might have true premises and a false conclusion. I will give the criticisms as directed specifically at the prototype argument. The first is that the argument is inconclusive because of redundancy. Either the universal premise is clear and agreed or it is not. If it is not then the argument is inconclusive because of an unclear premise. If the universal premise is clear then the argument could be propounded without the universal premise. So, the universal premise is redundant and the argument is therefore inconclusive.⁸³ Sextus' second argument seems to be roughly that if the conclusion is known in advance then the argument is useless and if the conclusion is not known then neither

⁸¹ Herbert Granger, "Heraclitus of Ephesus," *The Encyclopedia of Philosophy* 2nd ed. (New York, Macmillan, 2005), 316.

⁸² Sextus Empiricus, *Outlines of Scepticism*, transl. J. Annas and J. Barnes, (New York, Cambridge University Press, 2000), 112, 120. (The relevant passages are in Book II sections 164 and 196)

⁸³ *Ibid.*, 112.

are the premises, so the argument is useless. A similar criticism appears later:⁸⁴ the proponent of the [prototype] argument deductively confirms the conclusion with the premises but the universal premise is inductively confirmed by the conclusion. This argument would then be in the reciprocal mode where, according to Sextus, the best course of action is to suspend judgment about premises and conclusion.⁸⁵ (For Sextus, sentences are in reciprocal mode when each relies on the other for justification.)

Sextus' criticisms, I think, are transparently poor and if I responded to them I would say nothing beyond what a modern reader might think upon first reading them. Far from undermining my argument from authority, the fact that Sextus criticized the prototype argument speaks to its clear and apparent validity. He criticizes it as on par with the argument forms that are so clearly valid that they need no defense—modes ponens, modes tollens, and disjunctive syllogism—the Unprovables.

It is possible that my opponent could regard all of these authors as mistaken, e.g., all followed Aristotle in his mistake. Another alternative is that the arguments are ethymematic (in the modern sense). Consider the Heraclitus argument. Since we are attributing implicit premises to him anyhow, one might think we should attribute to him what he surely must have believed: there is anger. However, this move of attributing unspoken premises is much more difficult in the other cases. In each, the author is claiming validity for the argument within the context of a passage devoted to the characteristics of well-formed arguments. The arguments were the result of careful reflection and were intended to be the object of the same; in contemplative conditions such as those, unstated premises are unlikely.

⁸⁴ Ibid., 120.

⁸⁵ Ibid., 41.

Ambiguous Quantifiers Again?

The prototype argument is valid on PFL if the universal quantifier in the first premise is understood as being ontologically lightweight: either a substitutional quantifier or an objectual universal quantifier that ranges over the outer domain or that ranges over both domains. According to this response the mistake of thinking all prototype arguments are valid would be like the fallacy of accident—applying a general rule where it is inapplicable. Premise one is ambiguous between a heavy and light reading and only the light reading is valid.

As a rejoinder, I will present a version of an earlier argument: van Inwagen’s argument that the existential quantifier is univocal. If the universal quantifier has a light reading then so must the existential quantifier if the obvious relationships between the two are to hold. Furthermore, there must be uses of the light, existential quantifier (abbreviated ‘ \exists_L ’) that are equivalent to numerical claims (in the sense described in Chapter One), otherwise the light, universal quantifier would not range over count nouns. I will consider a special class of existentially quantified claims that are equivalent to numerical claims: those that paraphrase to ‘the number of Ps is exactly n.’ I am thinking of claims mainly of the form ‘there are exactly n Ps.’

The argument begins with the assumption that the existentially quantified claim that is alleged to be ambiguous is equivalent to a numerical claim. From there I argue that the numerical claim is not ambiguous, so the equivalent existential is not ambiguous either. In general, there is a problem with an argument like mine: it might be that just one reading of the allegedly ambiguous phrase is equivalent to the unambiguous paraphrase. For example, it obviously would not be proved that ‘bat’ is unambiguous merely because it is equivalent to the unambiguous ‘member of the order Chiroptera.’

There are two reasons why this is not a problem for the argument to follow. (1) If the light, existential quantifier were not equivalent to a numerical paraphrase (whereas the heavy existential was) then the light quantifier could not be used to range over countable categories, and so would be of no help in saving the prototype argument. (2) Furthermore, if the light existential quantifier were not equivalent to a numerical phrase there would be cases where back-substitution of the paraphrase into contexts where the alleged light existential quantifier was in use would be funny or false. However, there are no such cases. For example, taking a context where 'bat' allegedly means 'a ball-striking stick' and back-substituting 'member of the order Chiroptera' can take a truth to a falsehood. Consider, 'Carleton Fisk hit his famous extra-inning, '75 World Series home run with teammate Rick Burleson's *member of the order Chiroptera*.' In contrast, 'there are exactly x Ps' can always be replaced with 'the number of Ps is exactly x.' So if some reading of the existential quantifier can be replaced with the numerical claim then any reading can.

So, supposing that there is a heavy existential quantifier and a light existential quantifier, equivalence with a numerical claim would mean that there is a heavy and a light reading of

S: The number of Ps is exactly x.

Three tests seem to show that S is not ambiguous. None of the tests are conclusive (relying as they do on linguistic intuitions), but together they are convincing evidence that S is univocal. The first and third tests are general and based on Zwicky and Sadock's 1975 paper 'Ambiguity tests and how to fail them.'⁸⁶ The second test is discussion specific, and, as far as I know, originates with me.

⁸⁶ Arnold Zwicky and Jerrold Sadock. "Ambiguity tests and how to fail them." *Syntax and Semantics* 4, no.1 (1975): 1-36.

Test One: Contradiction

If S were ambiguous there would be a non-contradictory reading of

(C) The number of U.S. states is exactly fifty, and the number of U.S. states is exactly fifty-one.

However, intuitively, there is no such reading. So, S is not ambiguous.

Test Two: Addition

If S were ambiguous because of a non-overlapping ambiguity in ‘the number of’ (or in the numbers themselves) then there is no reason to think that the numbers would be addable. In general, I confidently propose (with a consistent reading of ‘the number of’):

If P and Q are disjoint and the number of Ps is exactly m, and the number of Qs is exactly n, then the total number of Ps and Qs is exactly $m + n$.

On the other hand, if nothing is both a number₁ and a number₂⁸⁷ then it would be very lucky if for some arbitrarily chosen P and Q,

The number₁ of Ps is exactly m and the number₂ of Qs is exactly n and either the total number₁ of Ps and Qs is exactly $m + n$ or the total number₂ of Ps and Qs is exactly $m + n$.

For example,

4. The number₁ of U.S. States is exactly fifty.
5. The number₂ of my gums is exactly forty-nine.

If (5) reports the (middle) age of my anesthetizing oral surgeon then both readings of (6) are false or nonsensical, and certainly they do not follow from the conjunction of (4) and (5).

6. The total number of U.S. states or my gums is exactly ninety-nine.

⁸⁷ This condition is necessary because if the numbers₁ are a subset of the numbers₂—e.g., rational numbers and real numbers—then the truth of the displayed sentence requires no good fortune at all.

Now consider the proposed heavy/light ambiguity with arbitrarily chosen P and Q. A defender of PFL might resort to that distinction to accommodate the truth of both (7) and (8).

7. The number of continents is exactly seven.
8. The number of lost cities of gold is exactly one.

It is perfectly coherent to conclude, on the basis of (6) and (7) that

9. The total number of continents or lost cities of gold is fifty-one.

A reasonable heavy/light defender would have to claim that (9) does not follow from (7) and (8) or accept that the light reading does. The former option is absurd; both senses of ‘number’ can be counted on fingers but not added?? The latter option might at first seem more promising. One could hold that the sufficient condition was unsatisfied (recall that the condition requires that nothing is both a number₁ and a number₂) and that the heavy numbers are a subset of the light numbers. However, if there are some things that are both heavy numbers and light numbers then every number is both a heavy number and a light one (what could be a light number but not a heavy one? fifteen?), and there is no real distinction.

Test Three: Conjunction Reduction

The test: take two sentences each with an occurrence of a purportedly ambiguous phrase and conjoin them. Reduce the result so that the purportedly ambiguous phrase appears only once, making grammatical modifications if necessary. If the new, reduced sentence could not be true (because the phrase is forced to do double duty) or is true only in a clever, literary sense (zeugma) then the phrase is ambiguous. For example,

She lost her keys.

She lost her cool.

Conjoined and reduced:

She lost her keys and her cool.

Since it is possible for the component sentences to be true and there be no sense on which the reduced sentence is true, ‘lost’ is ambiguous. An impossible, reduced sentence might not be necessary for ambiguity, but passing the test provides some evidence that a phrase is not ambiguous. It would be lucky for a reduced conjunction to be sensible given an ambiguity. So consider an example relevant to the purposes here:

The number of Tatooine’s suns is exactly two.

The number of Mars’s moons is exactly two.

Conjoined and reduced:

The number of Tatooine’s suns and of Mars’ Moons is exactly two.

The reduced sentence is plainly understandable;⁸⁸ it does not require any special literary interpretation to be understood.

Response (but there are ambiguous quantifiers):

Since my argument is that passing all three tests together is evidence of univocality, the criticism I will consider claims that there is an ambiguity (similar to, but not the same as the one the heavy/light proponents are after) that passes all three tests.

Start with a scenario:

The governor, a recently exposed liar, gives a speech defending his economic record. Outraged by the governor’s deception a heckler stands and reads loudly—loudly enough to make the governor inaudible to the crowd—quotations from the governor that are inconsistent with the themes of his current speech. Just before she is removed, she passes her page of quotations to her neighbor who also reads loudly from the document. After

⁸⁸ At least it seems to be to me. According to Larry Powers the sentence is “not English.” He suggests a different reduction: the number of Tatooine’s suns and Mars’ moons is the same, namely two.

the second heckler has been silenced, a group of three students stand and chant “fire the liar” until the governor ends his speech in frustration that his nasally voice cannot be heard above the chanting.

It would be appropriate to say that

(S) There were exactly two people who (alone) drowned out the governor.

And that,

(P) There were exactly three people who (together) drowned out the governor.

The first of these is true understanding the quantifier as a singular quantifier; the second is true on a plural understanding of the quantifier. (S) and (P) are each paraphrasable as numerical claims:

(S') The number of people who (alone) drowned out the governor is two.

(P') The number of people who (together) drowned out the governor is three.

If there were no non-contradictory readings of (C) above then there are no contradictory readings of the conjunction of (S') and (P'). Also, suppose (T') is translated from the obvious singularly quantified existential claim (There was exactly one person...).

(T') The number of people who threw tomatoes at the governor is three.

And (T') and (P') pass the addition test. It is acceptable to conclude from (T') and (P') that the total number of people who threw tomatoes or drowned out the governor is six. Furthermore, conjoin (T') and (P') and reduce in the manner of the conjunction reduction test.

The number of people who drowned out the governor and people who threw tomatoes at him is three.

The result is awkward, but is understandable. It requires no phrases to “do double duty” and it requires no special literary interpretation.

Consequently, by an argument similar to mine above, it could be argued that there is no ambiguity between singular and plural quantifiers—a seeming falsehood. In fact, the following four claims are apparently inconsistent:

- (PASS) The numerical paraphrases associated with (S) and (P) pass the Contradiction, Addition and Conjunction Reduction tests.
- (TEST) If numerical paraphrases pass the Contradiction, Addition, and Conjunction Reduction tests then there is no ambiguity of “the number of” in the numerical paraphrases.
- (INWAG) If there is no ambiguity of “the number of” in numerical paraphrases then there is no ambiguity of the quantifier in their quantified paraphrases.
- (AMBIG) There is a (singular/plural) ambiguity of the quantifier in the quantified paraphrases (S) and (P).

It will not do for me to accept the ambiguity and claim that it is not the heavy/light ambiguity that critics are after. That would require rejecting the test for ambiguity (TEST), the relationship between the quantified claims and numerical claims that my chapter-one, ontological-commitment argument (via van Inwagen) requires, or the seemingly clear results of the test applied to this instance (PASS). Given the apparent clarity of the test results, choosing this latter option would undermine the utility of the tests in general.

The problem is (AMBIG). There are two broad ways to maintain that quantifiers in English are not ambiguous between a plural and singular reading. One approach is to claim that all quantifiers are singular and that all seemingly non-distributive predicates take sets, sums or similar as arguments. A predicate F is distributive if whenever some things are F, each of them

is F.⁸⁹ For example, ‘are numerous’ is seemingly non-distributive since ‘ants are numerous’ does not entail that ‘each ant is numerous.’ Approach one claims that, despite appearances, numerosity is correctly predicated only of a single thing—a set with many elements, for example.

The second way of denying (AMBIG) takes all quantifiers to be possibly plural (in that they could take multiple domain elements as arguments) and includes singular quantifiers as a special cases. I favor this second route but offer no criticism of the first. If those who defend the first option are correct then the better for the argument in this chapter and for the semantics presented in Chapter Four. The argument below is neutral with respect to approach one or two.

If there were separate quantifiers in English, those that took individual arguments for distributive predicates and those that took multiple arguments for non-distributive predicates then we ought not to be able to make sense of a sentence with both kinds of predication but only a single quantifier. Such a sentence should seem incoherent, ungrammatical, or at least clumsy. On the contrary however, quantified sentences with mixed predication are perfectly coherent and natural.⁹⁰ Consider,

★ All of the students surrounded the building and sat down over time.

A plural quantifier is required if ‘surrounded the building’ is a non-distributive predicate, however ‘sat down’ (or hiding the quantification over times: ‘sat down over time’) is not required in ★ to take multiple arguments. In fact, the most natural reading of ★ is not that the students slowly and all-together sat down, but rather that, after surrounding the building, they each, possibly at different times, sat down. If ‘all’ must take either the plural-only or singular-only interpretation, then the only coherent reading of the sentence would be that the students who

⁸⁹ Thomas J. McKay, *Plural Predication* (New York, Oxford University Press, 2006), 5.

⁹⁰ This is similar to the conjunction reduction test and I am consequently open to a charge of circularity; I defend my ambiguity tests by appealing to one similar.

together surrounded the building, together sat down (slowly?). The ambiguity remaining in ★ must result from a distributive/non-distributive ambiguity in the predicate ‘sat down.’

For another example consider,

★★ Some isolated islands form a chain.

There are two readings of ★★ depending on whether ‘isolated’ is taken in a distributive or non-distributive sense. That is, the islands might be each isolated (and form a long, loose chain) or be together isolated (and form a short, dense chain). The distributive reading would be *unavailable* if ‘some’ were required to take a plural-only interpretation by the non-distributive ‘form a chain.’ This point is emphasized by a similar sentence,

★★★ Some small islands form a chain.

The only reasonable interpretation of ★★★ is that together some islands, each of them small, form a chain. It follows that ‘some’ does not have different readings in ‘some islands are small’ and ‘some islands form a chain.’⁹¹

This is sufficient to show, I think, that the singular/plural ambiguity does not apply to sentences because of ambiguous quantifiers (perhaps because of possible distributive/non-distributive reading of the predicates?). The quantifiers are univocal with respect to plurality. My preference for the view that quantifiers are univocally plural reveals a deficiency in the semantics I give in chapter four—which contains only singular quantifiers. Its applicability to the entire first order fragment of English is prevented by this fact (if it is a fact), unless one is willing to understand non-distributive predicates as distributive predicates taking sets or sums as arguments.

⁹¹ I am grateful to Eric Hiddleston for sentence ★★★. He suggested it to me in private correspondence.

Since the ambiguity tests are sound, it follows that in giving up the prototype schema, bivalent positive free logic gives up too much in order to save contingent (seeming) truths like ‘Sherlock Holmes is a detective’ and the Greeks worshipped Zeuss.’ In choosing a semantics to best represent natural language, our intuitions about the truth values of contingencies of fiction and mental states should not be valued above clear and long held intuitions about the most obviously valid arguments. Furthermore, appeal to heavy and light quantifiers cannot save both the contingencies and the prototype schema. Perhaps a trivalent approach can solve the problems of PFL and still escape the conclusion of the master argument from a priori existence claims.

Non-bivalent Positive Free Logic: Supervaluations

Supervaluational semantics (SVS) is a non-bivalent semantics designed to preserve the classical logical truths. First, I will explain SVS in detail. Next, I will develop and respond to what I believe is the point that most recommends it—the utility of its basic apparatus in handling several vexing philosophical problems—and then I will consider two criticisms of SVS as a general approach to philosophical problems, including the empty-names problem at hand. One of these criticisms is more successful than other, but neither is decisive. Finally, I will undermine the utility defense, by explaining away many of the intuitions that support it and by proposing disadvantages that counterbalance the remaining intuitive advantages.

Introduction to SVS

Supervaluational semantics (SVS) is three-valued, but preserves the classical tautologies. The neutral free logician must admit that $P \rightarrow P$ is not a logical truth, since P might contain an empty name. However, SVS nicely preserves logical truths by formalizing the intuition that $P \rightarrow P$ would be true *no matter what* the singular terms in P refer to. Bas Van Fraassen gave the

first formal account of SVS.⁹² The explanation here is adapted from his presentation.

Consider an interpretation **I** of language **L** with a domain **D** and a denotation function **d**. The function **d** takes predicates in **L** to extensions in **D** and constants in **L** to elements of **D**. Let **I** be non-standard only in that **d** is a *partial* function on the constants of **L**. The constants for which the denotation function is defined are given a referent by **d**. Variables are given referents by an assignment function **a**. An *x*-variant of **a** is a function that differs from **a**, if at all, only in the value it assigns to variable '*x*.' Truth values for **I** are assigned according to the following rules, wherein '*t*' represents a term that can refer—either a constant or a variable. The referent of '*t*,' if any, is represented by '**r**(*t*)'—which is either **d**(*t*) or **a**(*t*) depending on whether '*t*' is a constant or a variable. To reduce clutter in the remainder of this section, I dispense with inverted commas when mentioning symbols since no use/mention confusion is likely to arise.

(VA) If *t* refers: Pt is true iff $\mathbf{r}(t) \in \mathbf{d}(P)$ and false otherwise.

If *t* does not refer: Pt is neither true nor false (neither).⁹³

(VI) If both *t* and *t'* refer: $t=t'$ is true iff $\mathbf{r}(t)$ is $\mathbf{r}(t')$ and false otherwise.

If exactly one of *t* and *t'* refers: $t=t'$ is false.⁹⁴

If neither *t* nor *t'* refer: $t=t'$ is true iff *t* and *t'* are the same term, otherwise false.⁹⁵

(VC) If *A* is a complex formula: *A* is (super)true on **I** if all classical valuations give it T;

A is (super>false on **I** if all classical valuations give it F; otherwise *A* is neither.

A Classical Valuation is a function **c** that assigns T or F to every formula this way:

⁹² Bas Van Fraassen, 'Singular Terms, Truthvalue Gaps and Free Logic,' *Journal of Philosophy* 63 (1966): 481-95; Reprinted in *Philosophical Applications of Free Logic*, ed. Karel Lambert (New York, Oxford University Press): 82-97.

⁹³ To avoid indexing or a proliferation of variables, I have given only the unary predicate rule. The rule for atomic sentences with *n*-ary predicates is an obvious extension of (VA).

⁹⁴ Van Fraassen uses this rule to ensure that $\exists x(x=\text{Pegasus})$ comes out false. I believe this could be modified to "... $t=t'$ is neither' on the outer domain SVS in Lehman's 2002 "More free logic." That would give the desired result for the existence of Pegasus and also a more intuitive valuation for open sentences like ' $x=\text{van Fraassen}$.'

⁹⁵ Van Fraassen does not make this rule explicit but it gets the results he wants. It seems most natural to use classical valuations to get the same result, as on the outer domain SVS (see previous footnote), but he does not.

(CV) If formula A is true by (VA) or (VI) then $\mathbf{c}(A)$ is T. If A is false by (VA) or (VI) then $\mathbf{c}(A)$ is F. If A is neither by (VA) then $\mathbf{c}(A)$ is arbitrarily assigned either T or F.

(CVN) $\sim A$ is T iff A is F, otherwise $\sim A$ is T.

(CVD) $A \vee B$ is T iff either A is T or B is T

(CVE) $\exists x A(x)$ is T iff A is T for some x -variant of \mathbf{a} .

When atomic formulas contain no empty terms, the CV rules deliver classical results for complex formulas composed from the atomic ones. When an atomic formula A contains an empty term, (CV) arbitrarily assigns either T or F to A . So, there are at least two different classical valuations for any complex formula of which A is part. Suppose that on an interpretation \mathbf{I} , p does not refer but q does. Now consider: $Wp \vee Wq$. If p does not refer then Wp will be neither true nor false by (VA). Thus one classical valuation will assign it T and another will assign it F. If on \mathbf{I} , $\mathbf{d}(q) \in \mathbf{d}(W)$, then on both classical valuations (CVD) will assign T to the disjunction (since the right disjunct will be T by (CV)). However, if on \mathbf{I} , $\mathbf{d}(q) \notin \mathbf{d}(W)$ then the disjunction is T on one classical valuation and F on another. So, by (VC) the disjunction would be neither supertrue nor superfalse on \mathbf{I} .

If logical truth is truth on every interpretation then the formulas that are supertrue on every interpretation will be the classical logical truths.⁹⁶ Consider for example, $Wp \vee \sim Wp$. On interpretations where p does not refer, every classical valuation will assign T to either Wp or to $\sim Wp$. So the disjunction will come out supertrue by (VC). On interpretations where p does refer (VA), the CV rules, and (VC) will also make the disjunction supertrue. By defining logical truth as supertruth on all interpretations, SVS accommodates our intuition that ‘Pegasus has wings’ is

⁹⁶ Van Fraassen, “Singular terms, truthvalue gaps, and free logic,” 89.

neither true nor false while at the same time preserving the classical logical truths.

The cost of obtaining both desirables includes the loss of the (super)truth-functionality of the logical connectives. Suppose that on an interpretation \mathbf{I} , p does not refer. It follows that Wp , Vp and $\sim Wp$ are truth-valueless on \mathbf{I} . But then $Wp \vee \sim Wp$ and $Wp \vee Vp$ are both disjunctions of truth-valueless disjuncts. The former is supertrue on \mathbf{I} (and on all other interpretations) and the latter is neither true nor false on \mathbf{I} . So the wedge is not a function that takes values to unique outputs. With the exception of the (NVN) inputs, the connectives in van Fraassen's semantics have strong Kleene tables. The truth of $Wp \vee Wq$ on \mathbf{I} illustrates how a truth-valueless disjunct does not poison the disjunction as it does on the weak Kleene tables.

One could keep the truth functionality of the wedge by rejecting that supertruth is truth but by accepting that logical truth is supertruth on all interpretations. However, this maneuver has the obvious drawback of divorcing logical truth from truth.

The outer domain SVS presented by Lehmann⁹⁷ is more intuitive and is easier to adapt to other uses so I present that briefly too. Interpretations are partial as above. A completion \mathbf{I}' of an interpretation \mathbf{I} expands the domain to $\mathbf{D}+\mathbf{D}_o$ and includes a total function \mathbf{d}' that takes constants in \mathbf{L} to elements of $\mathbf{D}+\mathbf{D}_o$ and takes predicates in \mathbf{L} to extensions in $\mathbf{D}+\mathbf{D}_o$. When taking terms as arguments, \mathbf{d}' must have the same value as \mathbf{d} if the terms refer; if they do not refer, \mathbf{d}' must have values in \mathbf{D}_o . When taking predicates as arguments, $\mathbf{d}(P)$ must be the restriction of $\mathbf{d}'(P)$ to relations in \mathbf{D} . Assignments are total functions from variables to (just) \mathbf{D} , and similarly for x -variants.⁹⁸ The requirement that the range of assignment functions be in \mathbf{D} (rather than $\mathbf{D}+\mathbf{D}_o$) has the result that quantifiers range over only the inner domain not also over the elements added to serve as the referents of empty terms (the outer domain).

⁹⁷ Lehmann, "More free logic," 227-233.

⁹⁸ Lehmann describes completions for assignments, but I do not see that this is necessary; it is harmless to stipulate that assignments are total functions since there is no worry about non-referring variables.

We need only classical rules for valuations of formulas under completions and a rule for valuations of formulas under the incomplete interpretation **I**:

(VC) If A is formula: A is (super>true under **I** iff for each completion **I'**, A is true under **I'**; A is (super>false under **I** iff for each completion **I'**, A is false under **I'**; otherwise A is neither.

(VC') results in strong Kleene tables for the connectives.⁹⁹ Notice that $\exists x(x=p)$ will come out false if p has no referent since every assignment will assign to x an element of **D** and all completions will assign an element of **D₀** to p. Consequently, when the rule for identity makes atomic identities false when exactly one of the names refers (as do the rules for Lehmann, van Fraassen and Bencivenga) the existential sentence will be false on every assignment (since **a**(x) can never be the same as **d'**(p)).

An aside that will be useful to refer to later: To modify this result so $\exists x(x=p)$ will come out neither true nor false if p has no referent, one could change the (unstated) classical valuation rule for identity to the non-classical:¹⁰⁰

(VI) If both t and t' refer: $t=t'$ is true iff **r**(t) is **r**(t') and false otherwise. If either t or t' does not refer: $t=t'$ is (super>true if on every completion **d'**(t) is **d'**(t'), otherwise $t=t'$ is neither.

Without the modification mentioned in the aside, every interpretation that makes Wp true will be an interpretation on which $\exists x(x=p)$ is true (since p must refer for Wp to be true). However, the corresponding material conditional $Wp \rightarrow \exists x(x=p)$ is not logically true; if p lacks a referent then the conditional will be neither. (Some completions will assign p a referent **d'**(p) in **d'**(W) and some will assign a referent that is not in **d'**(W) so on some completions the antecedent will be

⁹⁹ Except for the NVN input which is indeterminate, as before.

¹⁰⁰ Brian Skyrms proposes a valuation rule that gives the same result as this one in "Supervaluations: Identity, Existence, and Individual Concepts," *Journal of Philosophy* 65, no.16, (1968) 479.

true and on some it will be false. For all completions the consequent will be false; so, by (VC') the conditional will be neither.)

This shows at once why SVS rejects Conditional Proof. For SVS the fact that B must be true on any interpretation on which A is true does not guarantee the truth of the conditional $A \rightarrow B$, since A's lacking truth value would result in the conditional lacking truth value as well (provided the consequent is false).¹⁰¹ I will return to this point; for neutral free logic also, the corresponding material conditional for many valid arguments is not a logical truth.

Utility Defense of SVS: Other uses

Support for SVS derives from its utility in solving puzzles while retaining classical logical truths (and in some cases also retaining intuitive, contingent truths). In addition to empty names, versions of SVS have been employed to handle the liar paradox,¹⁰² the problem of future contingents,¹⁰³ semantic over-determination,¹⁰⁴ the problem of the many,¹⁰⁵ multiply referring terms,¹⁰⁶ and vagueness.¹⁰⁷ In this document, I will briefly consider the latter three. The best way to respond to the utility defense, short of a kill-all criticism of any SVS approach, is to show that its seeming advantages are unnecessary and they are also counterbalanced (at least) by disadvantages.

Supervaluations and Vagueness

A supervaluational account of truth for sentences with vague predicates promises to

¹⁰¹ NFL rejects conditional proof for the same reason, except without the proviso that the consequent be false.

¹⁰² Bas Van Fraassen, "Presupposition, Implication and Self-reference," *Journal of Philosophy* 65 no.5 (1968): 136-152. The version I used is reprinted in Karel Lambert (ed.), *Philosophical Applications of Free Logic*, (New York: Oxford University Press, 1991), 205-221.

¹⁰³ See Richmond H. Thomason, "Indeterminist Time and Truth-Value Gaps," *Theoria* 36, no.3 (1970): 264-281.

¹⁰⁴ See Achille Varzi, "Supervaluationism and Paraconsistency" in *Frontiers in Paraconsistent Logic*, ed. Diderik Batens, Graham Priest and J.P. van Bendegem (Baldock, Hertfordshire, England: Research Studies Press, 2000), 279-297.

¹⁰⁵ Lewis, David, "Many but Almost One," in *Ontology Causality and Mind: Essays in Honor of D.M. Armstrong*, ed. Bacon, Campbell, and Reinhardt (New York: Cambridge University Press, 1993), 23-38.

¹⁰⁶ Greg Frost-Arnold, "Too Much Reference: Semantics For Multiply Signifying Terms," *Journal of Philosophical Logic* 37 (2008): 239-257.

¹⁰⁷ Kit Fine, "Vagueness, Truth and Logic," *Synthese* 30 (1975): 265-300.

resolve the Sorites paradox while preserving much of classical logic. In order to save classical logic but maintain that vague sentences are neither true nor false, the supervaluationists need a different account of completion than the positive free semantics described above. Here is one of the problems SVS aims to solve: Vague predicates like ‘is a tall structure’ can be applied both truly and falsely, however they are not precise enough to be applied truly or falsely in all cases. In borderline cases of application, there seems to be no semantic fact that makes the assertion true and none that makes it false. Consider,

(A) The Great Pyramid is a tall structure

uttered in a discussion of skyscrapers.¹⁰⁸ If (A) is neither true nor false then we need a three-valued semantics. One might be inclined to say that any sentence containing the vague predicate ‘is a tall structure’ is neither true nor false. However, the following all seem to be true:

(B) The Burj Khalifa is a tall structure.

(C) Either the Great Pyramid is a tall structure or it is not a tall structure.

(D) If the Great Pyramid is a tall structure the Washington Monument is too.

The first of these is a non-borderline case. The second and third contain only borderline cases, but express relationships that seem to hold despite the vagueness. In his proposal that SVS be used to model vagueness, Kit Fine calls these logical relationships between sentences with borderline cases ‘penumbral connections.’ Fine points out that no three-valued, truth functional account of the connectives can preserve penumbral connections like those expressed by (C) and (D).¹⁰⁹ SVS gives us a way to deny the truth (and the falsity) of (A) while accepting the truth of (B), (C), and (D).

Here is the general idea of the solution: Since vague predicates like ‘is a tall structure’

¹⁰⁸ I give a context of utterance only so I can mention that I will ignore complications that arise from the context sensitivity of vague predicates.

¹⁰⁹ Kit Fine, “Vagueness, Truth and Logic,” 269,270.

(abbreviate this predicate with ‘T’) do not completely partition the domain of discourse into an extension and an anti-extension, there will be an object g for which it is neither true that Tg nor true that $\sim Tg$. A *precisification* (or specification, or admissible valuation) is a way of making all the predicates of the language into a binary partition of the domain (and the n -ary relations a binary partition of the set of all n -tuples of the domain elements). Precisifications are subject to the following conditions: (a) all precisifications must preserve the status of non-borderline cases, e.g., if Tb is clearly true then it must be true on all precisifications. (b) All precisifications must preserve the status of general claims relating predicates (and relations) that are guaranteed by the meaning of the predicates (and relations), e.g., if ‘ $\forall x\forall y((Tx \ \& \ T_2yx) \rightarrow Ty)$ ’ is true by virtue of the meanings of the predicates—suppose ‘ T_2 ’ abbreviates the relation ‘is taller than’—then all precisifications must make it true. The precisifications are employed in the same way as the completions (or classical valuations) above; if a sentence is true (false) on all precisifications then it is supertrue (superfalse). If a sentence is true on some precisifications and false on others then it is neither. The truth criterion alone is sufficient to ensure the supertruth of (C). If logical supertruth is supertruth on all interpretations then (C) is logically supertrue. Condition (a) on precisifications ensures the supertruth of (B). Condition (b) ensures the supertruth—but not the logical supertruth—of (D).

Preserving penumbral connections is not the sole advantage of the supervaluational account of vagueness; it also resolves the Sorites paradox. The following premises seem to be true.

(E) A structure that is 2,700 feet high is a tall structure.

(F) For all n if an n inch structure is a tall structure then an $n-1$ inch structure is a tall structure.

Repeated applications (32,388 of them) of (F) give

(G) A one-foot high structure is a tall structure

which is clearly false. SVS retains the truth of (E) and the falsity of (G). Though the vagueness of the predicate ‘is a tall structure’ seems to require that (F) is true, SVS regards it as false. Since every precisification has a sharp boundary for ‘is a tall structure,’ it follows that (F) is superfalse. This is so despite the fact that for no n is it true that an n inch structure is tall and an $n-1$ inch structure is not. This abnormality is similar to one that we get with disjunction on SVS. ‘It is true that there exists something...’ can hold without ‘there exists something for which it is true that...’ Similarly, because SVS accepts the logical truth of $P \vee \sim P$ but not bivalence, a sentence of the form ‘either A or B’ can be true without its being true that ‘either it is true that A or it is true that B.’ We could describe this by saying that truth does not distribute over the existential quantifier or with disjunction.¹¹⁰

Not only does SVS give a motivated criticism of the sorites argument, it also explains why we might be taken in by the false premise. Some have criticized SVS by saying that it *does not* really explain why we are taken in by (F).¹¹¹ However, the failure of distributivity described in the previous paragraph offers an explanation.¹¹² We recognize that *there is no n for which it is true that n is the cutoff for T* , and we mistakenly take this to imply that *it is true that there is no n that is the cutoff for T* . As Rosanna Keefe describes it, we make a scope error similar to that in believing that *there is someone of whom it is true that they ought to do X* on the grounds that *it is true that someone ought to do X* .¹¹³ I agree with Graf Fara that this latter mistake is really one in

¹¹⁰ It is also true that on both SVS and NFL truth does not distribute over negation, since $\sim T(P)$ does not imply $T(\sim P)$.

¹¹¹ Delia Graf (Fara), ‘Shifting sands: an interest relative theory of vagueness,’ *Philosophical Topics* 28 (2000): 50-52.

¹¹² Rosanna Keefe, *Theories of Vagueness* (New York: Cambridge University Press, 2000): 185,186.

¹¹³ *Ibid.*

the scope of ‘ought’ not ‘it is true that.’¹¹⁴ However, Keefe’s example might be better taken as an analogy. If the merits of SVS motivate the failure of distributivity of truth over existential quantification, then it seems to be an additional merit that SVS proposes a subtly different and true alternative to the false (F).¹¹⁵

Supervaluations and the Problem of the Many

Peter Unger introduced what he called the ‘Problem of the Many’ by considering a cloud.¹¹⁶ The density of vapor decreases from the center of a cloud to its outer edges. Such a tapering results in a multitude of potential boundaries of the cloud. Since many of the potential boundaries will be equally good candidates, we have a puzzle: which, if any, of the candidates is the cloud? The options seem to be: say that all of the equally good possibilities are clouds or say that none of them are. Neither of those options is satisfactory since we were considering just one cloud. This problem is obviously not confined merely to clouds.

The supervaluational solution considers all precisifications of referential terms. Consider, the sentence,

★ That cloud is casting a shadow on the house.

If it is true for each of the equally good candidate referents of ‘that cloud’ that it is casting a shadow on the house then ★ is supertrue. Again, supertruth on an interpretation is truth on all precisifications. The precisifications here pick single referents from many possible in the domain, if our semantic practices fail to do so. On this account it will turn out true that there is just one cloud in the location of the overlapping candidates, for each precisification will pick one

¹¹⁴ See Delia Graff Fara, “Scope confusions and unsatisfiable disjuncts: two problems for supervaluationsism” in *Cuts and Clouds: Vagueness, Its Nature, and its Logic*, ed. Dietz and Moruzzi, (New York: Oxford University Press, 2010), 378,379.

¹¹⁵ If equivocation is the only fallacy then the explanation of a subtle but common mistake in reasoning is likely to rest on a subtle ambiguity.

¹¹⁶ Peter Unger, “The Problem of the Many,” *Midwest Studies in Philosophy* 5 (1980): 411-467.

precise boundary. As above, truth fails to distribute over the existential quantifier. There will be no one cloud of which it is true that it is the cloud casting a shadow on the house since different collections of vapor will cast the shadow on each interpretation. There is such a cloud but there is no answer to the question ‘which one?’ Lewis accepts this peculiarity and gives another analogy (with scope ambiguity): “I owe you a horse but there is no horse such that I owe you that horse.”¹¹⁷

Supervaluations and Multiply-Referring Terms

We considered SVS as a way of handling one form of reference failure—cases where no referent is available—but there is another way that reference can fail. There might be several candidates for denotation. Greg Frost-Arnold¹¹⁸ borrows an example from Joseph Camp.¹¹⁹ The owner of an ant farm mistakenly believes that there is exactly one large ant in the colony when, in fact, there are two. He tries to fix the referent of ‘Charley’ with the description ‘the big ant in the colony.’ Intuitively, ‘Charley is an ant’ is true as is ‘either Charley is eating or he is not.’ Here the supervaluationist considers disambiguations of multiply referring names. If a sentence is true on all disambiguations then it is supertrue. So, we get the desired result for both the contingent and tautological sentences above.

What was helpful for vagueness and the problem of the many now seems to be a curse. It comes out true on this account that *there is one and only one Charley*, since that sentence is true on all disambiguations. Frost-Arnold anticipates this criticism and offers a fix that results in $\exists x(x=\text{Charley})$ coming out superfalse.¹²⁰ Far from being an improvement, the fix seems to me to make matters worse. Frost-Arnold does give some reason for preferring his SVS account of

¹¹⁷ Lewis, “Many but almost one,” 29.

¹¹⁸ Frost-Arnold, “Too much reference,” 241.

¹¹⁹ Joseph Camp, *Confusion*, (Cambridge: Harvard University Press, 2001).

¹²⁰ Frost-Arnold, “Too much reference,” 250.

multiple-reference to NFL; he thinks our logic would be needlessly impoverished by treating Charley just like Santa Claus. We would lose useful information. Consider $9=y^2$. We could say that y is odd since both of the candidate solutions have absolute value 3. Contrast with $y=1/0$. We cannot say in this case that y is even or odd since $1/x$ is undefined when $x=0$.¹²¹

Criticisms of SVS

I will present two criticisms that apply generally to all of the SVS accounts presented: (1) non-distributivity of truth, and (2) the absurdity of a counterfactual theory of truth. I will say why I do not think the first criticism is successful, and why I think the second one is. I will also give specific criticisms of each application of SVS where possible. Then for each application, I will try to defend an NFL-style approach.

My real target is SVS for empty names. However, the primary benefit of SVS is the utility of the general idea of completions. Before presenting the two general criticisms of the SVS approach I will say why I take my response to bivalent PFL as insufficient to undermine SVS for names.

Reconsider the schema of the prototypically valid argument:

(P) All A are B; c is A; so, c is B.

SVS for names invalidates the following argument,

(Q) $\forall x((Ax \vee \sim Ax) \rightarrow Bx); Ac \vee \sim Ac; \therefore Bc$.

On an interpretation where 'c' does not denote an element of the domain and where B includes every element of the domain, (Q) has true premises (premise two is logically supertrue) and a false conclusion. (Q) is an instance of (P). Allowing an invalid instance of (P) is unacceptable (by my argument above), so SVS seems to be as badly off as bivalent PFL.

¹²¹ Ibid., 251, 252.

I am moved by this argument but I can see why someone else might not be.¹²² The force of the argument against PFL is the obvious intuitive validity of the schema (P). The more like (P) a natural language argument looks, the more absurd it is to embrace a semantic system that counts that argument as invalid. However, (Q) only appears to be an instance of (P) given a background in symbolic logic, and natural language counterexamples to the validity of (Q) either seem to depart far enough from schema (P) or contain elements that otherwise undermine a clear intuition that the argument is valid. For one example,

Everything that is either tall or not tall is physical.

Camelot is either tall or not tall.

So, Camelot is physical.

Even if the first premise could be defended (which it would need to be for a proper counterexample to validity), the disjunctive antecedent of that premise is dissimilar to that of the corresponding premise in the most obviously valid prototype arguments. Consider an example without a disjunctive premise:

All self-identical things are existent things.

Camelot is self-identical.

So, Camelot is an existent thing.

This argument has the structure of schema (P), but now relies on several confusing and controversial components that might interfere with our clear intuitions about (P): that self-identity is a property, that SVS must make $c=c$ true, and that being an existent thing is a property.

Since this criticism of SVS is unconvincing, I will present the first of the general criticisms to SVS.

¹²² In fact, Larry Powers showed me this result as the key step in a reductio ad absurdum of my argument against PFL.

Distribution of Truth

On every SVS, truth fails to distribute over either disjunction or the existential quantifier.

Suppose ‘p’ does not refer. Then on an SVS for names,

$$\star \quad T(Wp \vee \sim Wp)$$

but not,

$$\star\star \quad T(Wp) \vee T(\sim Wp)$$

An SVS for vague predicates gives the same result if we suppose that ‘p’ does refer but is a borderline case of W. Furthermore, on SVS for vagueness truth does not distribute over the quantifier. I will give the example first in English,

It is true that there is a height at which a building is tall and below which it is not tall.

However it does not follow on SVS for vagueness that,

There is a height for which it is true that a building at that height is tall and below which it is not.

In symbols,

$$True (\exists n((Hb = n) \rightarrow Tb) \cdot ((Hb = n - 1) \rightarrow \sim Tb))$$

But not,

$$\exists n True (((Hb = n) \rightarrow Tb) \cdot ((Hb = n - 1) \rightarrow \sim Tb))$$

There is no corresponding distribution failure over the existential quantifier on SVS for names since all systems are specially formulated so that $\exists x(x = p)$ is untrue when p does not refer (Ermanno Bencivenga¹²³ and Lehman’s Outer Domain SVS make it false;¹²⁴ Brian Skyrms¹²⁵ makes it neither true nor false when p is non-referential).

¹²³ Ermanno Bencivenga, “Truth, correspondence, and non-denoting singular terms,” *Philosophia* 9 no. 2 (1980): 219-229.

¹²⁴ Lehman, “More free logic,” 228.

¹²⁵ Brian Skyrms, “Supervaluations: Identity, Existence, and Individual Concepts,” *Journal of Philosophy* 65, no.16, (1968): 479.

On SVS for problem of the many, truth fails to distribute over both disjunction and the quantifier. For the first pair of sentences let X be a volume between the maximum and minimum possible volumes of the cloud.

It is true that either the cloud has volume greater than X or volume less than or equal to X . However, it does not follow that,

Either it is true that the cloud has volume greater than X or it is true that the cloud has volume less than or equal to X .

All precisifications of the cloud will give it a volume either greater than X or less than or equal to X . However, not all precisifications will have volume greater than X , and not all precisifications will have volume less than or equal to X . So, the first sentence can be true while the second is not.

Failure of distributivity over the existential quantifier is shown similarly:

It is true that there is a molecule on the boundary of the cloud.

But this, for reasons similar to those above, does not follow,

There is something such that it is true that it is a molecule on the boundary of the cloud.

On SVS for multiple-reference truth does not distribute over disjunction. The sentences \star , and $\star\star$ above are sufficient to show non-distributivity. On an interpretation where 'p' is multiply referential and one referent of 'p' has W and the other does not, \star holds but $\star\star$ does not.

Whether or not distributivity fails for the quantifier depends on whether $\exists x(x = p)$ is true or untrue if 'p' multiply refers. If the sentence is true, as it is for Hartry Field in the case of multiple reference,¹²⁶ then truth will not distribute over the quantifier. There will not be an x such that it is true that x is identical to p . Frost-Arnold, who intentionally avoids making

¹²⁶ Field, Hartry, "Theory change and indeterminacy of reference", *Journal of Philosophy* 70, no.14 (1973): 462-481.

$\exists x(x = p)$ true,¹²⁷ maintains distributivity over the quantifier (so far as I can tell).

My proposal is that the oddness of non-distributivity is not sufficient as a criticism of SVS. I count it as the core of SVS (for names) that LEM ($\forall A: A \vee \sim A$) but not BIVAL ($\forall A: TA \vee T\sim A$).¹²⁸ Merely relying on the intuition that $T(A \vee B)$ necessitates $TA \vee TB$ begs the question against SVS, since it would immediately imply that LEM necessitates BIVAL (by substitution of $\sim A$ for B).

So an argument is required that $T(A \vee B)$ necessitates $TA \vee TB$, or similarly that LEM ($A \vee \sim A$) implies BIVAL ($TA \vee T\sim A$). I think that there are no persuasive arguments of that sort that do not beg the question against SVS. At least, they beg the question in the sense that they will rely on a single premise or an inference that SVS takes to be objectionable; primarily I suspect the culprit will be BIVAL as it is in the examples considered here. The rejection of that premise or inference might be so intuitively absurd that the argument is a reductio of SVS. However, I do not think one is likely to find a premise or inference whose rejection is more absurd than the rejection of BIVAL (which is insufficient as a criticism of SVS)—since it will either be BIVAL that justifies the premise or inference or else it will be some less intuitive principle that does the justifying. Below I offer two examples of arguments that seem to derive BIVAL from LEM (or show more generally that truth distributes over disjunction) but that really are no stronger than the intuition for BIVAL.

Note: I give an argument in a later chapter that LEM implies BIVAL. That argument is not intended as a criticism of SVS. It is intended for someone who is already convinced of NFL so it should not secretly rely on bivalence (and does not). Construed as an argument against SVS it would come down to this principle: *There can be no false sentence obtained by substitution of*

¹²⁷ Frost-Arnold, "Too much reference," 245.

¹²⁸ For the remainder of the chapter I use 'T' to represent a truth predicate.

a false component for a neutral component in a true sentence. That is a principle that is implied by bivalent semantics or trivalent semantics with either weak or strong Kleene tables. It is not a principle that carries so much intuitive weight that it could be used in a persuasive argument against SVS. The prima facie persuasive arguments seem to rely on BIVAL—at least that is my argument in this section.

Here is an example of a proof where each line is justified by the line above and where each justification seems to be acceptable:

1. $A \vee B$
2. $T(A \vee B)$ since P necessitates TP
3. $T \sim (\sim A \cdot \sim B)$ by DeMorgan's
4. $\sim T(\sim A \cdot \sim B)$ since T~P necessitates ~TP
5. $\sim (T\sim A \cdot T\sim B)$ since T distributes over conjunction
6. $\sim T\sim A \vee \sim T\sim B$ by DeMorgan's
7. $\sim\sim TA \vee \sim\sim TB$ since T~P necessitates ~TP
8. $TA \vee TB$ by Double Negation

The objectionable step is from 6 to 7. That T~P necessitates ~TP does not license substitution. To suppose that ~T~A necessitates ~~TA is to assume bivalence. I count ‘~TA necessitates T~A’ as the inferential form of Bivalence for its obvious similarities to BIVAL even though Material Implication is not valid for necessitation. Substitute ~A for A and the rule that justifies 7 becomes ‘~T~A necessitates ~~T~A,’ which after applying Double Negation, is just the inference form of Bivalence: ~TA necessitates T~A.

Sayward's Or-Elimination Argument

Or-elimination is the rule that allows derivation of C given AVB and both a subproof

from A to C and another from B to C . Van Fraassen and SVS cannot accept the schema $A \leftrightarrow TA$, if it is understood to license substitution. Van Fraassen explains the intuitive appeal of the schema in a way that the NFL appreciates: The argument from A to TA is valid, as is the argument from TA to A . However, that co-necessitation does not license substitution. Otherwise, $A \vee \sim A$ would immediately imply $TA \vee T\sim A$. Charles Sayward argues that SVS is still in a pickle since every interpretation with A has TA and every interpretation with $\sim A$ has $T\sim A$. So, any proof system should allow derivation of TA from A and $T\sim A$ from $\sim A$. Consequently, acceptance of $A \vee \sim A$ and application of or-elimination yield BIVAL.¹²⁹

- | | | |
|----|-------------------|---|
| 1. | $A \vee \sim A$ | LEM |
| 2. | A | Assume for or-elimination |
| 3. | TA | From 2 by a model-theoretic justification |
| 4. | $TA \vee T\sim A$ | Addition from 3 |
| 5. | $\sim A$ | Assume for or-elimination |
| 6. | $T\sim A$ | From 5 by a model-theoretic justification |
| 7. | $TA \vee T\sim A$ | Addition from 6 |
| 8. | $TA \vee T\sim A$ | By or-elimination from 1,4,7 |

As Timothy Day points out, SVS rejects to the use of Or-elimination. It cannot be used, he says, “unless it can be shown that the disjuncts are not both gap.”¹³⁰ To corroborate my point, that Sayward’s criticism is no stronger than the intuition for bivalence, consider the argument that Sayward is trying to improve upon: a constructive dilemma with LEM and conditionals from the disjuncts of LEM to the appropriate disjuncts of BIVAL as premises and BIVAL as

¹²⁹ Charles Sayward, “Does the law of excluded middle require bivalence?” *Erkenntnis* 31, no.1 (1989): 134. (I have embellished Sayward’s argument.)

¹³⁰ Timothy Day, “Excluded middle and bivalence,” *Erkenntnis* 37, no.1, (1992): 94,95.

conclusion.¹³¹

$$(1)A \vee \sim A, (2)A \rightarrow TA, (3) \sim A \rightarrow T \sim A \therefore (4) TA \vee T \sim A$$

SVS rejects the conditionals.¹³² So the problem is that or-elim amounts to using conditional proof to recover the conditionals and then using (unobjectionable) constructive dilemma. However, the use of conditional proof—or more generally of dischargeable assumption—requires or assumes bivalence.

The easy way to see this is to consider an interpretation that makes A neither true nor false; on such an interpretation TA is false. So on that interpretation, $A \rightarrow TA$ is neither true nor false. Any interpretation which has A will have TA . So, assuming A is true should allow derivation of TA , which, given Conditional Proof, would allow derivation of $A \rightarrow TA$. Consequently, the rejection of bivalence implies the rejection of conditional proof (provided TA can be stated in the language of A).

This might be unwelcome help to SVS since the brief metalinguistic argument in the paragraph above relies on several components that SVS rejects in the object language: reductio ad absurdum, contraposition, and assertability of TA . Since my aim is not ultimately to defend SVS but to reduce the distributivity criticism to a criticism no stronger than appeal to bivalence, I am satisfied with this argument whether or not van Fraassen is.

Incidentally, Both SVS for names and SVS for vagueness seem to lose conditional proof in the object language, even if ‘ TA ’ is unstateable in the object language. For names the counterexample is $Pa \rightarrow \exists x(x = a)$. The consequent is true on every interpretation that makes

¹³¹Van Fraassen anticipates this argument in “Singular terms, truth-value gaps, and free logic,” 95,96. (pagination from Lambert, *Philosophical Applications of Free Logic...*)

¹³² Insisting on the conditionals is also insisting on Bivalence. For example, $A \rightarrow TA$, if accepted as true on all interpretations would be equivalent by contraposition to $\sim TA \rightarrow \sim A$. (Contraposition is valid for conditionals on SVS but not for necessitation.) Consequently $\sim TA$ would necessitate $\sim A$ which in turn necessitates $T \sim A$. So it would be a requirement for accepting $A \rightarrow TA$ that one accept the inferential form of BIVAL, that $\sim TA$ necessitates $T \sim A$.

the antecedent true but the conditional is neutral on an interpretation where Pa is neutral because the consequent is false on such an interpretation (by fiat for both van Fraassen and Bencivenga).¹³³ The counterexample to conditional proof on SVS for vagueness is $Pa \rightarrow D(Pa)$ where $D(Pa)$ means Pa is true on every precisification.¹³⁴ Intuitively, $D(Pa)$ reads ‘Definitely Pa .’ Any interpretation on which the antecedent is true must be one where $d(a) \in d(P)$ on every precisification of P , and consequently $D(Pa)$ will be true. So $D(Pa)$ is deducible from the assumption that Pa . However, when a is a borderline case of P then the conditional will have a neutral antecedent and a false consequent.

In addition to the fact that arguments for distributivity seem to share a weakness, there is at least some motivation for non-distributivity.

Linguistic Evidence for Non-Distributivity

There is some linguistic evidence that truth does not distribute over disjunction. Consider the following pair of questions,

- (Q1) Is it true that either the Tigers are in Anaheim or Baltimore tomorrow?
- (Q2) Is it true that the Tigers are in Anaheim tomorrow or is it true that the Tigers are in Baltimore tomorrow?

The explicit syntactic difference between the questions is the very distinction that SVS says matters. (Q1) might be represented as ‘ $T(A \vee B)$?’ and (Q2) could be symbolized ‘ $T(A) \vee T(B)$?’

‘Yes’ would be an appropriate answer to (Q1) but it would be a strange answer to (Q2); ‘Baltimore’ would be an appropriate answer. This is not a clincher for the distinction, of course, since answering ‘yes’ to (Q1) and ‘neither’ to (Q2) would be very strange. However, there are some interesting things to note about the pair: knowing the appropriate answer to (Q2)

¹³³ Van Fraassen “Singular terms, truthvalue gaps, and free logic,” 93. Also, Lehmann “More free logic,” 229.

¹³⁴ Timothy Williamson, *Vagueness* (New York: Routledge, 2000), 151,2.

implies knowing the appropriate answer to (Q1). In contrast, one might be able to confidently answer (Q1) ‘yes’ but answer (Q2) with ‘I cannot say.’

I do not want to make too much of the distinction since (Q2) seems to be a way of asking ‘which of A or B?’ rather than asking ‘ $T(A) \vee T(B)$?’ (for which a simple affirmative would be an appropriate answer). Notice though, that what SVS says is that in some cases disjunctions (and existential generalizations) can be true without there being an answer to the question of which one it is—because specifically, $T(A \vee B)$ might hold without $T(A) \vee T(B)$. In this example the reason one might answer the first query without being able to answer the second is a lack of knowledge rather than a lack of determinate answer. However, someone defending the distinction between the two questions should be heartened; we use (Q2) for the which-question, not (Q1). If $T(A \vee B)$ and $T(A) \vee T(B)$ trivially co-necessitated each other then it would be more likely that (Q1) would be ambiguous between the two questions (Is the disjunction true? Which disjunct is true?) Or (Q2) could be used to ask either question (Is one of the disjuncts true? Which disjunct is true?)— but that is clearly not the case. In other words, why should ‘ $T(A) \vee T(B)$?’ but not ‘ $T(A \vee B)$?’ be a way of asking ‘which one?’ if the nature of truth makes $T(A) \vee T(B)$ equivalent to $T(A \vee B)$?

In addition to lending credence to the claim that the $TA \vee TB$ is prima facie not equivalent to $T(A \vee B)$, the contrasting questions (Q1) and (Q2) suggest another line of defense for SVS. There are several operators that do not distribute over disjunction: knowledge, justification, and necessity. Knowledge that $A \vee B$ does not imply either knowledge that A or knowledge that B. Having justification for $A \vee B$ does not imply having justification for A or having justification that B. Necessity of $A \vee B$ does not imply necessity of A or necessity of B. SVS might argue by analogy for the non-distributivity of truth.

If I am right that the arguments for non-distributivity do not carry the day then we need another argument against the SVS approach. A better argument is that SVS is inconsistent with our intuitive understanding of truth.

The Absurdity of a Counterfactual Theory of Truth

Ermanno Bencivenga suggests that truth can be thought of as the verdict of an ideal experiment that checks a statement for correspondence with reality. When names do not refer, truth is a verdict of a *mental* experiment: would the statement be true on all completions?¹³⁵ Keefe defends against the objection that precisifications should not play a role in truth because our predicates are not precise (often the point of a predicate is to be imprecise¹³⁶). She says that quantification over all precisifications captures the meaning of the predicates even if individual precisifications do not.¹³⁷

Lehmann wonders why truth ought to be understood in counter-factual terms when typically it is regarded as correspondence to fact.¹³⁸ Keefe provides a response to the analogous criticism:

The justification for taking the supervaluationary approach to truth-conditions and employing precisifications is that it results in a successful theory of vagueness in all the ways already listed.¹³⁹

Lewis might respond (as he did to a different criticism), “so it is [peculiar]. But once you know the reason why you can learn to accept it.”¹⁴⁰

The successes of SVS (of any kind) are two-fold: it allows for truth value gaps and

¹³⁵ This description of the view comes from Lehmann, “More free logic,” 233.

¹³⁶ Michael Dummett, “Wang’s Paradox,” *Synthese* 30, no.3/4 (1975): 312.

¹³⁷ Keefe, *Theories of Vagueness*, 190.

¹³⁸ Lehman, “More free logic,” 233.

¹³⁹ Keefe, *Theories of Vagueness*, 192.

¹⁴⁰ Lewis, “Many but almost one,” 29.

preserves many classical logical truths (all tautologies and many predicate logical truths).¹⁴¹ I argue that NFL does as well at allowing truth value gaps, and the tautologies are not worth saving. In a later chapter I will provide argument for abandoning LEM. Also, in this chapter, I will try to account for the intuition that all classical logical truths (CLTs) are logical truths (LTs). If these responses hold, then there is no positive reason to accept a counterfactual theory of truth. Only Bencivenga, to my knowledge, has defended the counterfactual theory of truth with an argument other than the utility defense.

Bencivenga's Mental Experiments

Lehmann criticizes SVS (at least Bencivenga's version) as endorsing a counterfactual theory of truth. Fact, Lehmann says, should not be decided on the basis of what is contrary to fact.¹⁴² In "More Free Logic", Lehmann presents Bencivenga's argument for the counterfactual theory of truth. The argument Lehmann presents in defense of a counterfactual theory of truth is *consistent* with what Bencivenga says in "Truth, Correspondence, and Non-Denoting Singular Terms." However, the argument is not even *prima facie* compelling and *must not* be what Bencivenga had in mind. I will present a more charitable reconstruction and then offer a criticism of it. Here's the interesting portion of the argument as Lehmann presents it:

(1) Where terms refer..., truth or falsity may be identified with the outcome of an ideal practical experiment that compares what the sentence says with the way the world is...(2) In most cases where terms do not refer..., such practical experiments are out of the question; but we may nonetheless identify truth or falsity with the outcome of mental experiments...that assign such terms non-existent referents.¹⁴³

Of course, the mere possibility of doing so is no evidence that we ought to do so. Astonishingly, Bencivenga himself puts his argument the same way:

[Even on the classical schema] the ascertainment of the truth or falsity of a given (atomic)

¹⁴¹ For vagueness, problem-of-many, and multiple reference it also preserves seemingly obvious contingent claims; there is no corresponding benefit for SVS for names.

¹⁴² Lehmann, "No input no output logic," 152.

¹⁴³ Lehmann, "More free logic," 232.

sentence can always be regarded as the result of some sort of practical experiment. If we want to know whether Pt is true, we must look around the world for t and decide whether or not it is a PIf practical experiments are in principle excluded, are not there other kinds of experiments that we can perform? In particular, is it not possible (and useful) to perform *mental* experiments? In other words, what *if Pegasus existed?*¹⁴⁴

Bencivenga must mean to defend SVS with more than the mere possibility of its being correct. I see two possibilities for charitably strengthening the argument. Bencivenga could justify the step from practical experiments (when terms refer) to mental experiments (when they do not) by (1) analogy or by (2) a unifying principle. Consider the analogy first:

The reference-only cases are similar to the cases where non-reference is possible in that experiments (for ascertaining truth) are possible. Since experiments are useful in reference-only cases, they are also useful in possible non-reference cases. Furthermore, if they are useful then the underlying semantics that makes them most useful (that is SVS) is likely to be correct.

Despite its apparent improvement, this argument only benefits from an ambiguity in ‘useful.’ If ‘useful’ is taken to mean ‘successful without fail’ then the last inference is strong; if mental experiments succeed without fail then a semantics that implies their infallibility is likely to be correct. However, with such a strong conclusion, the analogy portion of the argument is weakened by the dissimilarities between the cases. The experiments possible in the reference-only cases differ from the experiments in the possible non-reference cases in two obvious ways. The first is that the former experiments are practical and the latter are practical and mental. The second key (and even more obvious) difference is that the later cases permit reference failure. Both of these differences are plainly relevant to whether experimental ascertainments of truth are infallible. The similarities between cases (the mere *possibility* of experimental ascertainments of truth) are not sufficient to outweigh the dissimilarities.

¹⁴⁴ Bencivenga, “Truth, correspondence, and non-denoting singular terms,” 224

The analogy seems more plausible if we understand ‘useful’ in the more modest sense: ‘frequently successful.’ Experiments remain useful (in the modest sense), even if cases of reference failure are permitted, because the case under experiment *might not be* cases of reference failure. However, on that reading, the final step of the argument is no longer strong. The more fallible the mental experiments are, the less likely it is that the underlying semantics is one that makes them infallible.

Now consider the second alternative for charitably interpreting Bencivenga:

It is a desired or necessary characteristic of a theory of truth that the ascertainment of truth can be regarded as the result of some kind of experiment. So, when practical experiments are not possible in cases of reference failure we must resort to another kind of experiment and mental experiments (of the counterfactual variety) are the most reasonable of the alternatives.

The general premise seems true. If it were not, truth might be in-principle unascertainable. However, when the established experimental method is inapplicable in some case, it is not obviously appropriate to search for another method. It is often appropriate to conclude that truth is inapplicable.

The supervenientist might respond that she has provided a disjunctive method that is widely applicable, but the desirability of wide application would need to be justified. We do not value semantics that apply truth values to linguistic items like commands and questions. SVS must resort to the desire to save intuitive (logical) truths like LEM. However, this is just the utility defense again, and the neutral free logician has all she needs to counter that. She has a lucid reason for abandoning LEM (and other tautologies which imply it), and an explanation of its intuitive truth. In fact, mental experiments provide another explanation for the faulty (in the

eyes of NFL) intuition that classical tautologies are logical truths.

The counterfactual mental experiment *is* handy when considering matters of logic: assessing validity for instance. Furthermore, it has its utility (in the fallible sense) in evaluating sentences for logical truth. It is commonplace to ignore a sentence when checking for logical truth. Think about the mental process of checking the following formula to see if it is a tautology: $\sim A \vee (\sim A \rightarrow B)$. I ignore B and consider the possibilities for A. In a sense, this is supposing B has a truth value. This process is sufficient for determining the logical truth of the sentence when B does have truth value. Supposing that B has truth value does not seem like a good way to get the right answer if B does not have truth value. This counterfactual process does not lead us astray in safe and commonplace cases where all names refer, so we are accustomed to employing it successfully. Consequently, because counterfactual mental experiments are fairly reliable, classical tautologies seem intuitively logically true. The right way to think about the SVS counterfactual test is as a way of explaining our (faulty) intuitions about LEM, not as a way of saving LEM.

If Bencivenga's argument fails to provide support beyond the utility defense of SVS then it only remains to criticize the utility defense for SVS.

Accounting for CLTs with Corresponding Arguments (a criticism of the utility defense)

One major criticism of NFL is that it fails to preserve the classical logical truths (CLTs) which are intuitively logical truths simpliciter (LTs). It is a perceived advantage of SVS that it preserves as logical truths all CLTs. I do not think anyone claims (or should claim) that it is *intuitively* clear that *all* CLTs are LTs. If there is justification for that universal claim it must come from generalization from a collection of CLTs that are intuitively LTs. I do not think the class is very large; I would include the law of excluded middle, non-contradiction and perhaps a

few others. For many other CLTs that appear to be LTs simpliciter, the justification for their LT status seems to be their status as CLTs. These of course, on pain of circularity, could not be used as a justification that all CLTs are LTs. Surely, opponents of neutral free logic regard the group (of CLTs that are intuitively LTs) to be larger than I do. However large the collection may be, what follows is an argument that I intend to undermine the force of the intuitive appeal of CLTs as LTs. I argue that there are corresponding valid inferences available to account for the intuitive status of *any* CLT as an LT. This removes the appeal of supervenience as a method for preserving the CLTs. It is not worth the cost if the intuitive status of CLTs as LTs can be otherwise accounted for.

Truly accounting for intuitions seems to require telling a justified psychological story about how one acquires the mistaken intuitions. My aim is less lofty than to provide evidence of a certain psychological process. I think of my goal as merely criticizing or undermining an argument from intuition. An argument for SVS based on the intuitive appeal of logical truths would have the general form:

Many have the intuition that there are truth value gaps and that many CLTs are LTs.

So, there are truth value gaps and many CLTs are LTs.

SVS is consistent with both truth value gaps and with many CLTs being LTs.

So, SVS is correct.

The first step in this argument relies on the unstated premise that many would not have the reported intuitions if they were not true. To undermine this argument, it is not necessary to demonstrate that the unstated premise is false; it is sufficient to cast doubt on the premise. And, giving a plausible alternative is all that is necessary to cast doubt. If a detective believes on the basis of a hunch that a suspect is guilty, it is sufficient to undermine that argument to give a

plausible story of the hunch's origin that is unrelated to its veracity. For example, if we find out that the suspect has previously insulted the detective's mother then that is sufficient, without any psychological experimentation, to undermine what little evidence the hunch provided.

To account, in this limited way, for the intuitive appeal of a CLT by means of corresponding arguments, two requirements should be met: the correspondence between the sentence and the arguments should be obvious, and the validity of corresponding arguments should be at least as intuitive as the LT status of the sentence. I will call a set of arguments that satisfies the first requirement with respect to a sentence the 'corresponding arguments' for the sentence. For a sentence S to have a set of corresponding arguments A , it must be intuitively clear that S is a CLT if and only if the members of A are all valid.

A classical logical truth (CLT) is *inferential* if it can be (obviously) represented as a set of valid corresponding arguments or argument schemata whose validity is obvious or intuitive (at least) to the extent that the CLT is obviously or intuitively a CLT.

Here are a few examples that illustrate inferentiality:

$A \rightarrow A$	$A/\therefore A$
$\sim((A \rightarrow A) \rightarrow \sim(B \rightarrow B))$	$A/\therefore A$ and $B/\therefore B$
$\forall x(Fx \rightarrow Fx)$	$Fx/\therefore Fx$

I will try to prove that all CLT are inferential. The argument to follow is by induction on sentence length. I will consider only conditionals, negations and universal quantifiers to simplify the proof. One might think this is a problem because conversion of a logical truth to a limited set of symbols adds complexity in some cases (for example conjunctions to conditionals) but I will ignore it. Disjunctions to conditionals are hardly more complex. Also, on the especially concerning case of conjunctions to conditionals: notice above the drastic reduction in complexity

in the second example going from a ‘conditionalized’ conjunction to the set of inferences. I will say a little more about disjunctions after the basis step below. First consider the more complicated induction step.

Induction Step: Suppose that all CLT of length N are inferential. It follows that all CLT of length $N+1$ are inferential. Consider a CLT of length $N+1$.

If it is of the form $\alpha \rightarrow \beta$, then clearly the validity of $\alpha \therefore \beta$ is no less obvious than the status of $\alpha \rightarrow \beta$ as a CLT. Also, (famously and) obviously the corresponding argument is valid iff $\alpha \rightarrow \beta$ is a CLT.

If the CLT of length $N+1$ is of the form $\forall x \alpha$ then there are two options: either x is not free in α or it is. If x is not free in α then α is itself a CLT and, since it is shorter than $\forall x \alpha$, by supposition it is inferential. Prefixing α with a vacuous quantifier cannot make it more intuitively a CLT, nor can it make the relationship with corresponding arguments less intuitive. If x is free in α then substituting a constant ‘ c ’ for x in $\alpha(x)$ will yield a CLT. Since $\alpha(c/x)$ is length N , it is inferential. So there is a set of corresponding arguments or schemata whose validity is at least as obvious as the CLT status of $\alpha(c/x)$. Replacement of all occurrences of ‘ c ’ in that set with an unused variable will not reduce the clarity of the inferences. Consequently, noting the validity of the members of the resultant set (containing at least one schema) will be no less obvious than noting that a CLT is obtained by universally quantifying over the free variable in α .

If it is of the form $\sim \alpha$ then there are three cases to consider. Since no atomic sentences are classically logically false, α is not atomic. So α must be a negation, a conditional or a universal quantification. If it is a negation then $\sim \alpha$ is of the form $\sim \sim \beta$ and is clearly equivalent to β which has length $N-1$. Since by supposition β is inferential, it follows that $\sim \alpha$ is too.

If α is a conditional then $\sim\alpha$ has the form $\sim(\beta \rightarrow \gamma)$. If $\sim\alpha$ is a CLT then β is a CLT and $\sim\gamma$ is a CLT. Since β has length $N-1$ and $\sim\gamma$ has length N , by supposition, both are inferential. Since each can be represented as a set of intuitively valid corresponding arguments or schemata, $\sim\alpha$ can be represented by the union of those sets of obviously valid arguments. This joining of sets representing β and $\sim\gamma$ is no more complex (possibly less complex if the sets share members) than the conjunction of β and $\sim\gamma$ themselves. So the validity of the inferences in the resultant set will be at least as intuitive as the CLT status of $\sim\alpha$. Also, the union of sets corresponds to $\sim\alpha$ since the sets correspond to β and $\sim\gamma$.

If α is universally quantified then $\sim\alpha$ is of the form $\sim\forall x\beta$. Either x does not appear free in β or it does. If it does not then $\sim\beta$ is a CLT, which (having length N) is inferential (by supposition). Since without a free x , $\sim\beta$ is clearly equivalent to $\sim\forall x\beta$, it follows that $\sim\alpha$ is inferential. If x does appear free in β then (α is really non-vacuously, existentially-quantified, so) the situation is complicated and requires a further subdivision of cases: three cases where β is a negation, the case where β is a conditional, and the case where β is universally quantified.¹⁴⁵

Consider first the trivial case where β has the form $\sim\sim\gamma$ (or where $\sim\alpha$ has the form $\sim\forall x\sim\sim\gamma$). Since β is plainly equivalent to γ , the consecutive double negation can be deleted, reducing sentence length. Since $\sim\forall x\gamma$ has length $N-2$, it is inferential (by supposition) and consequently, so is $\sim\forall x\beta$.

Second consider the case where β has the form $\sim(\gamma \rightarrow \delta)$ (or where $\sim\alpha$ has the form $\sim\forall x\sim(\gamma \rightarrow \delta)$). This is really the case of an existentially quantified conditional. Thus, $\sim\alpha$ is

¹⁴⁵ One brief aside from the argument: no universally free logic (including SVS) preserves any (non-vacuously) existentially quantified logical truths since the empty domain is a permitted counterexample. So there is no obvious advantage gained over SVS in accounting for their intuitive LT status. However, my criticism of SVS's intuitive support has wider applicability (and is less ad hoc) if it can account for all CLTs.

equivalent to $\forall x\gamma \rightarrow \sim\forall x\sim\delta$ which has an obvious counterpart inference: $\forall x\gamma/\therefore\sim\forall x\sim\delta$ which is valid iff $\sim\alpha$ is a CLT. The equivalence of CLTs *as written* is not immediately obvious (at least to me) until the CLTs are reckoned with the existential quantifier and disjunction. Here is the simple chain of equivalences that justifies the contention that the equivalence is intuitively clear:

$$\begin{aligned} & \sim\forall x\sim(\gamma \rightarrow \delta) \\ \star & \quad \exists x(\gamma \rightarrow \delta) \\ & \quad \exists x(\sim\gamma \vee \delta) \\ & \quad \exists x\sim\gamma \vee \exists x\delta \\ & \quad \sim\forall x\gamma \vee \exists x\delta \\ \star\star & \quad \forall x\gamma \rightarrow \exists x\delta \\ & \quad \forall x\gamma \rightarrow \sim\forall x\sim\delta \end{aligned}$$

Note that when x does not occur free in either γ or δ then the quantifier construction preceding it in $\star\star$ is vacuous and can be eliminated for increased clarity (this is equivalent to “pushing in” the existential quantifier via the well-known rules of passage). Each step in the above chain is obvious. Of course that on its own does not guarantee that *all steps together* reveal an obvious equivalence;¹⁴⁶ however, another point does suggest that fact. The equivalence between \star and $\star\star$ is natural for evaluating existentially quantified conditionals.

Existentially quantified conditionals—even those that are not very complex—are surprisingly counterintuitive. Consider for example, Raymond Smullyan’s Drinking Principle and its “dual version:” (1) there exists a person such that if he drinks then everyone drinks, and

¹⁴⁶ Although, there is a Sorites-style argument for the conclusion that it does: you cannot go from an obvious equivalence to a non-obvious equivalence by means of a single obvious step...

(2) there exists a person such that if anyone drinks then he does too.¹⁴⁷ Both are CLTs (when represented in the obvious way in predicate logic) despite their initial appearance as preposterous.¹⁴⁸ It is difficult to see with clarity the CLT status of $\exists x(Dx \rightarrow \forall yDy)$ without “pushing in” the existential quantifier to the antecedent as a universal quantifier. Similarly for the “dual principle,” to evaluate $\exists x(\exists yDy \rightarrow Dx)$ it is most natural to push the existential quantifier to the consequent—a move justified by the equivalence between \star and $\star\star$.

The move from \star to $\star\star$ is even simpler when reckoned model-theoretically in (quasi-)English. There is something such that if it satisfies γ then it satisfies δ . So, either something fails to satisfy γ or something satisfies δ . So, if everything satisfies γ then something satisfies δ . The inferences are equally clear in the reverse direction, so \star is obviously equivalent to $\star\star$. Two applications of the quantifier exchange definition $\sim\forall x\sim \leftrightarrow \exists x$ (which could be dropped had I lengthened this induction argument by considering both quantifier cases) yield the obvious equivalence in question. Since $\sim\alpha$ is intuitively equivalent to the inferential $\forall x\gamma \rightarrow \sim\forall x\sim\delta$, it follows that $\sim\alpha$ is inferential.

Now the third case, where β is of the form $\sim\forall z\gamma$ (or where $\sim\alpha$ has the form $\sim\forall x\sim\forall z\gamma$), is really of the form $\exists x\forall z\gamma$ and I will handle it with (and in the same way as) the final case below, where $\sim\alpha$ has the form $\sim\forall x\forall z\gamma$ (or $\exists x\exists z\sim\gamma$).

Before that, consider the case where β is of the form $\gamma \rightarrow \delta$ (or where $\sim\alpha$ has the form $\sim\forall x(\gamma \rightarrow \delta)$). In less restricted language, this is the case where $\sim\alpha$ is an existentially quantified

¹⁴⁷ Raymond Smullyan, *What is the Name of this Book?* (Englewood Cliffs N.J.: Prentice Hall, 1978), 209-211.

¹⁴⁸ It seems to follow from this observation that accounting for the intuitive status of existentially quantified conditionals as CLTs might not demand the effort I devote to it here; however a complete induction proof requires the effort.

conjunction. The existential quantifier does not distribute both ways over conjunction, so for this case, $\sim\alpha$ cannot be reduced to a conjunction of two shorter sentences in order to make use of the induction hypothesis. As in the case of a universally quantified conditional, a special maneuver is called for.

The sentence $\sim\forall x (\gamma \rightarrow \delta)$ is a CLT iff the argument $\forall x (\gamma \rightarrow \delta) / \therefore \sim\forall x (\gamma \rightarrow \delta)$ is valid (call this argument the “corresponding reductio inference” of the CLT). In fact, *any* sentence S is a CLT iff $\sim S / \therefore S$ is valid. Suppose S is a CLT, then $\sim S$ cannot be true; so the argument could not have true premises and an untrue conclusion. Suppose that the argument could not have true premises and an untrue conclusion, then $\sim S \rightarrow S$ is a CLT (but not an NFLT). Since $\sim S \rightarrow S$ is classically equivalent to S , it follows that S is a CLT. It remains to show that the validity of the reductio inference is at least as intuitive as the CLT status of $\sim\alpha$.

If it is intuitively clear that $\sim\forall x (\gamma \rightarrow \delta)$ is satisfied in every classical model then it must be intuitively clear that $\forall x (\gamma \rightarrow \delta)$ is not satisfied in any classical model. That in turn implies that it is obvious that the assumption of $\forall x (\gamma \rightarrow \delta)$ implies contradiction, and consequently that the reductio inference is valid. All of the converse inferences follow with equal clarity.

It might be noticed at this point that if $\sim\alpha$ in this case is inferential because of the argument $\alpha / \therefore \sim\alpha$, then this fact might have been used to significantly shorten the forgoing argument that all CLT are inferential. There are two reasons why the maneuver is only appropriate now. (1) Often the simplest method for establishing that an existentially quantified conjunction is a CLT is reductio ad absurdum, and the $\sim S / \therefore S$ argument is really a reductio inference. Also, (2) the complexity of existentially quantified conjunctions that are CLTs makes the corresponding inference at least as obvious as the CLT status of $\sim\alpha$; that is not obviously so for the more transparent CLTs and their corresponding reductio inferences (e.g., $A \rightarrow A$).

It is important to note that until this step, the induction proof given above could serve as a recipe for finding corresponding valid inferences for CLTs. However, in many cases of existentially quantified conjunctions (or negations of universally quantified conditionals) there is an even simpler corresponding valid argument (or set of arguments) than the reductio inference.

I give some examples below:

$$\exists x[(Pa \rightarrow Pa) \cdot (Qb \rightarrow Qx)] \quad \{(Pa/\therefore Pa), (Qb/\therefore \exists x Qx)\}$$

$$\exists x[(Pa \rightarrow Px) \cdot (Pb \rightarrow Px)] \quad (Pa \vee Pb)/\therefore \exists Px$$

Here is one intuitive CLT that seems to require the reductio inference:¹⁴⁹

$$\exists x[(\forall y Py \rightarrow Px) \cdot (\exists z Qz \rightarrow Qx)]$$

Two cases remain: $\sim a$ is of the form $\sim \forall x \sim \forall z \gamma$ and $\sim a$ has the form $\sim \forall x \forall z \gamma$. Both are cases of nested quantifiers where the leading quantifier is existential. Again, the reductio inference is sufficient to account for the intuitive status of $\sim a$ as a logical truth. The argument is similar to the case of an existentially quantified conjunction. I present it again for just the first of the nested quantifier cases.

If it is intuitively clear that $\sim \forall x \sim \forall z \gamma$ is a logical truth, then it must be clear that it is satisfied in every model and consequently that $\forall x \sim \forall z \gamma$ is not satisfied in any model. So it is clear that $\forall x \sim \forall z \gamma$ implies a contradiction and consequently that $\forall x \sim \forall z \gamma / \therefore \sim \forall x \sim \forall z \gamma$ is valid.

Now for **the basis step**: The only atomic CLT is $c = c$. The corresponding argument is $\exists x(x = c) / \therefore c = c$. The correspondence is clear. If the corresponding argument had a model with a true premise and an untrue conclusion then there would be a classical model where $c = c$ was untrue. I do not think there is a question over whether the intuitive status of the argument is as clear as the intuitive LT status of $c = c$. If it is true that c is, then it must be that c is itself. The

¹⁴⁹ In addition to being a CLT, it is also, if we ignore the empty domain, a neutral free logical truth by virtue of its containing no individual constants.

question will be over whether the corresponding inference could be falsely contributing to the intuition that $c = c$ is an LT.

In fact, this sort of corresponding argument could be used to account for any CLT, so why is it especially appropriate to use it here? Because identity is what we use to express existence. A thing cannot be itself without being, nor can a thing be without being itself. Even if this is unconvincing and $c = c$ is left as an exception, the inductive argument still has substantial force without it.

The basis step would begin then with length one. No negations of atomic sentences are CLT and no open or closed atomic sentences can be universally quantified to yield a CLT. So, the only length one CLT is $A \rightarrow A$. That is clearly inferential; the corresponding argument is $A/\therefore A$.

Finally, it follows that all CLT are inferential.

An initial reaction might be to worry that my conditions for inferentiality are not sufficient to fully account for the intuitions that CLTs are LTs. Do I need a psychological argument that we are liable to confuse the corresponding argument for the CLT? Consider LEM. When mentally evaluating it, is anyone (in fact) influenced by the inference $A; \text{ so, } A$? My aim is not to give a psychologically well-defended theory of the origin of our intuitions about LTs. I wish only to provide a plausible possible route to our intuitions about LTs that do not require the intuitions to be correct. But is my inferential account of LEM even plausible?

I think it is plausible. The following reasoning in defense of an obviously truth-valueless disjunction is somewhat compelling:

- If it were true that ‘Lewis Carroll saw the mome raths outgrabe’ then it could not be untrue that ‘Charles Dodgson saw the mome raths outgrabe.’

- So, either Charles Dodgson saw the mome raths outgrabe or Lewis Carrol did not see the mome raths outgrabe.

Someone initially skeptical of the conclusion could plausibly come to accept it if she became convinced of the inferential claim in the premise. If this kind of reasoning from a claim about the validity of an argument to the necessary truth of a disjunction is appealing in this case (whether or not the premise is true) then the same reasoning could be persuasive in the case of LEM. However, it could be the kind of persuasive argument that rarely occurs to someone who is considering the LT status of a disjunction. Again, I think it is plausible that it commonly occurs to someone to reason that way. It is natural to read an inclusive disjunction as saying at least one of two components is true. It is also natural then to suppose that one of the disjuncts is false and next determine whether it follows that the other disjunct must be true.

Saving Contingent Truths

There is one remaining aspect of the utility defense of SVS to deal with. The three versions of SVS that are not of particular interest to me all have an advantage that SVS for names does not have; they all preserve some intuitive *contingent* truths.

The Burj Khalifa is tall.

That cloud is casting a shadow on the house.

Charley is an ant.

If saving contingent truths with vague predicates, imprecise objects, and multiply referring terms is important enough to justify a counterfactual theory of truth, then a counterfactual theory of truth cannot be dismissed out of hand. If the benefits of SVS in some contexts persuade us that truth is counterfactual in those contexts, then the oddity of a counterfactual theory of truth cannot be used against SVS for names.

My response relies on two claims, which I regard as uncontroversial. (1) No obviously valid inferences should be sacrificed to save contingent, intuitive truths. (1) The evidence provided by the intuitive appeal of contingent truths is undermined if we are forced to accept contingent, intuitive falsehoods too. The first consideration undermines Frost-Arnold's SVS for multiple reference. However, Field's SVS for multiple reference avoids that criticism. The final consideration is sufficient to undermine the other SVS accounts that all preserve contingent, intuitive truths.

First, recall that on Arnold's SVS for MR, simple existentially quantified claims with multiply referring terms come out false (because they are equivalent to 'there is exactly one...'). That feature means that my criticism of PFL is sufficient to undermine Frost-Arnold's SVS semantics for multiple reference. The following prototypically valid argument is obviously valid, but is not on Arnold's account:

Everything that is an ant is something.

Charley is an ant.

So, Charley is something.

The obvious symbolization is

$$\forall x(Ax \rightarrow \exists y(y = x))$$

Ac

$$\therefore \exists y(y = c)$$

On an interpretation where 'Charley' is multiply referring and both referents are ants, the premises are true and the conclusion is false (because for Frost-Arnold $y = c$ is anti-satisfied by every sequence since c is undefined¹⁵⁰). Obviously, a logical system should not sacrifice an

¹⁵⁰ Frost-Arnold, "Too much reference," 249.

obviously valid inference for the sake of preserving some intuitive contingent truths.

Field's SVS semantics for multiple reference avoids this problem (at least Arnold's formalization of Field's proposal does¹⁵¹), by maintaining the logical supertruth of the conclusion. Since the conclusion is equivalent to 'there is exactly one Charley,' his version of SVS shares with SVS for problem-of-the-many and for vagueness that there are contingent, intuitive truths that SVS counts as false. Consequently, appeal to our intuitions of contingent truths (as support for a counterintuitive counterfactual theory of truth) is untrustworthy.

The intuitive falsehoods that SVS makes true come in two related types: (1) the problem that SVS was intended to solve cannot be truly stated, and (2) SVS is inconsistent with intentional or obvious imprecision.

SVS is Too Successful

Consider,

'Tall' is a vague predicate.

There is no single best cloud candidate.

'Charley' refers to two ants.

All three of these are intuitively true, yet come out false on every appropriate precisification. So, according to SVS there is no problem to be solved by SVS.¹⁵² That is a drawback if the support for SVS rests on its facility in dealing with those problems. David Lewis attributes this criticism of his solution to the problem of the many to a conversation with Saul Kripke.¹⁵³ Keefe considers it as a criticism of SVS as an account of vague sentences. In fact, she expands the

¹⁵¹ Ibid., p.245

¹⁵² Frost-Arnold suggests that this is a problem for SVS for non-referring names (p. 246). It is not—at least not for either of the two versions that I presented. In fact, Bencivenga takes great pains to avoid having $\exists x(x=\text{Pegasus})$ come out as supertrue when 'Pegasus' does not refer. For example, see his *Logic, Bivalence, and Denotation* (Atascadero CA; Ridgeview, 1991) .95-97.

¹⁵³ Lewis, "Many but almost one," 29,30.

criticism. Descriptions of the SVS solution seem to be false on SVS; her example is ‘F can be made precise in many ways.’¹⁵⁴ Lewis points out that a key component in the generation of the problem of the many is that there are many equally qualified candidates for being the cloud. However, on each sharpening there is just one cloud; the tie is broken. So, it will be supertrue that there is a single best cloud candidate. Consequently, SVS will have been employed to solve a problem that cannot be generated by the lights of SVS.

Lewis’s response is to employ the SVS rule for truth when it is helpful but not when it is not. ‘Fanatical Supervaluationism’ would apply the rule to all statements. He says,

The rule should instead be taken as a defeasible presumption. What defeats it, sometimes, is the cardinal principle of pragmatics: the right way to take what is said, if at all possible, is the way that makes sense of the message. Since the supervaluationist rule would have made a hash of our statement of the problem, straightaway the rule was suspended.¹⁵⁵

Keefe agrees that it is sometimes appropriate to apply the SVS rule in ordinary circumstances, but not when reflecting on precisifications and vague predicates. However, she does not endorse Lewis’s pragmatic approach. Besides sacrificing generality, it might, she says, permit a Sorites paradox in cases where the supervaluationist rule is not permitted. The difference between approaches is that for Keefe, use of vague predicates occurs in the object language, but mention of them occurs in a meta-language.

Keefe considers the statements, ‘F has borderline cases’ and ‘F can be made precise in many ways’ (among others). Opponents of SVS can claim that on each precisification both statements are false. The first is a description of the problem for which SVS was employed and the latter is a component of the SVS solution. The first she suggests interpreting as ‘ $\exists x(\sim T(Fx))$ ’.

¹⁵⁴ Keefe, *Theories of Vagueness*, 186.

¹⁵⁵ Lewis, “Many but almost one,” 30.

$\sim T(\sim Fx)$ ' where T is the meta-language truth predicate that is applied according to the SVS rule.¹⁵⁶ That SVS rule renders borderline cases of F neither true nor false so the meta-linguistic sentence is true. Similarly, precisification talk can be handled with the truth predicate at the meta-level. Even if this escape to the meta-level avoids the discomfort (for the first and third example), there are intuitively untrue, object-level variants that SVS regards as true.

There is a height at which buildings above that height are tall and those below are not tall.

There is a single best cloud candidate.

There is exactly one Charley.

These sentences are all intuitively untrue, but there are similar sentences that seem even more obviously untrue.

Intentional Imprecision

Consider the following examples,

- There is an exact (to infinite decimal places) range of volumes and dimensions such that any volume within that range is roughly the size of a fist and such that any volume outside that range is not roughly the size of a fist.¹⁵⁷
- The American Midwest has precise (to infinite decimal places) coordinate boundaries.
- There is exactly one John Doe.

Intuitively, each of the displayed sentences is untrue. The imprecise term in each is *intentionally* imprecise; there are useful linguistic purposes for having a vague-making adverb 'roughly', for having imprecisely defined geographical regions, and for having multiply referring names. The fact that the corresponding SVS system makes each of the displayed sentences true (for

¹⁵⁶ Ibid., 188.

¹⁵⁷ This example is inspired by McGee and McLaughlin's incredulity at what is true on Williamson's epistemic account of vagueness; It is absurd, they say, that there is a cutoff to infinite decimal places for the predicate 'is roughly the size of a breadbox.' Vann McGee and Brian McLaughlin, "Review of Vagueness," *Linguistics and Philosophy* 21, no.2 (1998): 221.

vagueness, problem of the many, and multiple reference, respectively) means that the SVS rule for truth prevents us from using imprecise terms *even when we intend to be using them*. Implausible as that is, on its own, intuitions about the truth values of contingent sentences are not overwhelming evidence against any of the versions of SVS. However, they are, I believe, sufficient to counterbalance the intuitive pull of the contingent truths that SVS is intended to save.

Since both bivalent and supervaluational positive free logic free logic sacrifice much more than they gain, they are not feasible candidates for answering the challenge of the master argument. It remains only to consider rejection of (the logical truth of) the schema $a = a$.

CHAPTER THREE: NEGATIVE FREE LOGIC AND BIVALENCE

Broadly speaking, there are two ways to reject an instance of ‘ $a=a$ ’ where ‘ a ’ does not refer: one could regard it as neither true nor false or simply as false. The neutral free logician prefers the first of these options and the negative free logician prefers the latter. R. M. Sainsbury succinctly explains the negative free logician’s preference:

Negative free logic is motivated by two thoughts: (i) a true simple predication refers to something and predicates a property which that object possesses (the idea extends to any n -ary simple sentence); (ii) falsehood is failure of truth.¹⁵⁸

If Sainsbury is right then it seems as if all simple predications (and as a result, all truth-functional combinations of them) are either true or false. In this chapter, after a brief introduction to negative free logic (NgFL) and some *prima facie* criticisms, I will show that three arguments in favor of negative free logic all come down to bivalence (for simple predications) and then argue that there are not good justifications for bivalence. I will also argue that for the neutral free logician, the real issue is the rejection of Law of Excluded Middle (and rejection of Bivalence is secondary). I will consider several arguments against accepting (as true) all instances of it. This last section will also serve as a further criticism of supervaluational semantics, since SVS preserves LEM as logically true.

The large part of this chapter is an objection to the motivations for bivalence and LEM. That on its own of course, is not evidence against negative free logic. So, after a brief description of NgFL, I begin the chapter with three *prima facie* problems for negative free semantics that are intended to at least send the negative free logician seeking for a justifying reason to hold the view.

¹⁵⁸ Sainsbury, R.M., *Reference without Referents* (New York: Oxford University Press, 2005), 66.

Introduction to Negative Free Logic

The form of negative free semantics that takes the least departure from classical logic holds that denotation functions can be partial and also that atomic sentences, including identity statements, simple predications, and n-place relation statements are false if they contain a non-referring name. Also, negations of false atomic sentences are true. Consequently, the classical logical truths are preserved. However, some of the inferences are lost; Universal Instantiation is restricted to referring terms, and Existential Generalization is permitted only from referring terms. For example, ‘Pegasus is not identical to Pegasus’ does not entail ‘something is not identical to itself.’ The foremost advantage of negative free semantics is that negative existentials match our intuitions. For example, ‘Santa Claus does not exist’ comes out true. The appropriate symbolization is $\sim\exists x(x = s)$. Since ‘s’ does not refer, $\alpha = s$ is false for all domain elements α assigned to x by the variants of the assignment function (or to the name replacing x by variants of the denotation function). Consequently, $\exists x(x = s)$ is false and the target sentence is true.

There are three sources to consider on negative free logic. Rolf Shock,¹⁵⁹ Tyler Burge¹⁶⁰ and Richard Sainsbury¹⁶¹ all endorse a version of NgFL. Perhaps there are really only two; Sainsbury adopts Burge’s NgFL,¹⁶² however Burge does not have name scope device and Sainsbury offers one (without employing it himself).¹⁶³ Shock, Burge, and Sainsbury all agree on the minimal requirements of NgFL mentioned above.¹⁶⁴ Other possible divergences are treatment of empty domain, and allowing descriptions as terms. Schock and Burge explicitly

¹⁵⁹ Schock, Rolf, *Logics Without Existence Assumptions*. Stockholm, Almqvist & Wiksell. 1968

¹⁶⁰ Tyler Burge. “Truth and singular terms”. *Noûs*, 8, no. 4, (1974): 309-325.

¹⁶¹ Sainsbury, *Reference without Referents*. 2005.

¹⁶² *Ibid.*, 178,9.

¹⁶³ *Ibid.*, 70, 196.

¹⁶⁴ See: Schock, *Logics Without Existence Assumptions*, 40. Burge, “Truth and singular terms,” 313. Sainsbury, *Reference without Referents*, 70.

allow the empty domain. For Schock, all atomic sentences and existentially quantified sentences are false on the one empty domain interpretation, and universally quantified sentences are true (even vacuously quantified atomic sentences).¹⁶⁵ Burge does not say how he expects his truth-rules to work on the empty domain. It is not immediately clear how his rules will work since figuring it out requires knowing how the quantifiers in the meta-language work in the absence of sequences.¹⁶⁶ I discuss the empty domain in chapter four, so I forgo discussion of it here. It is worth considering here the name scope indicator, since that device is suggested by NgFL and by no other free logic.

Sainsbury eschews appeal to a name/negation scope ambiguity to explain the seeming falsity of ‘Vulcan is not more than 1000 miles in diameter.’ For Sainsbury, ‘not’ always takes large scope, and the intuition that the sentence is false comes from an alternate reading (without a negation) wherein the sentence attributes to a supposed referent of ‘Vulcan’ the property of being less than or equal to 1000 miles in diameter.¹⁶⁷ Earlier,¹⁶⁸ Sainsbury had suggested appeal to a name scope indicator in order to distinguish, for example (mine), between two readings of

A. Vulcan is not a planet.

On one reading the sentence says,

B. It is not the case that Vulcan is a planet.

For NgFL, that sentence is true if ‘Vulcan’ is non-referential. On another reading the sentence says,

C. Vulcan is such that it is a non-planet.

For NgFL with a name-scope device, (C) accounts for the intuitions (if any) that (A) is false.

¹⁶⁵ Schock, *Logic Without Existence Assumptions*, 41.

¹⁶⁶ Burge, “Truth and singular terms,” 315.

¹⁶⁷ Sainsbury, *Reference without Referents*, 198.

¹⁶⁸ *Ibid.*, 70.

On a neutral free logic, there is no need to appeal to wide and narrow scope names (or wide and narrow negation). If there were a name-scope distinction in English (which needed to be reflected in the first-order semantics) then, incredulously, the sentence ‘Al did not introduce Barb to Cal’ would be eight-ways ambiguous (because there are eight distinct combinations of the three names that could take large scope relative to the negation). Furthermore, a semantics with the name scope indicator allows an invalid interpretation of the following, obviously valid argument:

For all x and y, if x is seen by y then y sees x.

Al does not see Barb.

So, Barb is not seen by Al.

If ‘Al’ has no referent, has small scope in the second premise, and has large scope in the conclusion then the argument could have all true premises and a false conclusion. This possibility is counter to the intuition that the argument is unequivocally valid.

Sainsbury’s own criticism of a variable-scope ‘not’ in English is that on the assumption of one, ‘Vulcan does not exist,’ would be ambiguous between a true and false reading.¹⁶⁹ A neutral free logician agrees with Sainsbury that the simple non-existence claim does not have readings with differing truth-values. Of course, NFL does not agree with his assertion that the sentence is unambiguously false.

This departure from the minimal requirements of NgFL is, in my opinion, not worth the cost. In any case, the criticisms of NgFL that follow are all criticisms of the minimal departure from classical logic common to *all* NgFL.

Three Criticisms of Negative Free Logic

First, here is a simple argument against negative free logic, mentioned by Sainsbury who

¹⁶⁹ Ibid., 197.

cites David Wiggins and Mark Heller as proponents.¹⁷⁰

★ It is not the case that Vulcan is identical to Vulcan.

★★ Vulcan is distinct from Vulcan.

Since ‘is identical to’ and ‘is distinct from’ are both simple predicates, ★ is true and ★★ is false on a negative free logic (assuming that ‘Vulcan’ does not refer). However, ★ and ★★ are synonymous so they cannot differ in truth value. In defense of NgFL, denying the synonymy of ★ and ★★ is not *absurd*, nor is denying that ‘is distinct from’ is a simple predication. However, neither response is especially appealing either. At least the simple argument sends us looking for a justification for negative free logic.

The second criticism is an argument familiar from an earlier chapter. Here is an argument that seems to have the logical form of a prototype argument (see chapter two):

Everything that is not material is immaterial.

Pegasus is not material.

So, Pegasus is immaterial.

For the negative free logician it is possible for this argument to have all true premises and a false conclusion, since it has the form:

$$\forall x(\sim Mx \rightarrow Ix)$$

$$\sim Mp$$

$$\therefore Ip$$

Any model where $\mathbf{d}(I)$ contains all domain elements not in $\mathbf{d}(M)$ makes premise one true. Of those models, the ones where ‘p’ has no referent makes premise two true and the conclusion false.

¹⁷⁰ Ibid., 69.

The negative free logician sometimes appeals to an ambiguity between a small and large scope negation. Here perhaps the negative free logician could say that premise two is ambiguous between the false ‘Pegasus is non-material’ and ‘not: Pegasus is material.’ However, the objection continues, the universal quantifier (ranging as it does over domain elements) requires interpreting the negation in the antecedent as having small scope—because there could be no substitution instances that would make the large scope negation reading of the antecedent true and the small scope negation reading of the antecedent false. So, the intuition of the argument’s validity comes from the small-scope negation reading of premise two. On the large-scope negation reading the argument is invalid.

I disagree that that the small-scope negation reading of the antecedent is a requirement; if one distinguishes between large and small scope negation readings of ‘Vulcan is not a planet’ then one ought to make the same distinction with respect to ‘it is not a planet’—where ‘it’ is best represented by a free variable. Even if I am right and the scope response fails, this criticism of NgFL is not a decisive refutation.

Because of the leading negation in premise two, the prototype argument above departs slightly from the most obviously valid natural language prototype arguments; consequently the objection is not, in my opinion, conclusive. However, the loss of the argument is still sufficient to warrant an intuitively stronger justification than the prototype argument. Perhaps bivalence is it!

The third prima facie problem for negative free logic is the loss of another obviously valid inference:

All and only things identical to Groth are identical to Gardner.

So Groth is identical to Gardner.

If ‘Groth’ fails to refer then the conditional is vacuously true for all substitutions, but the conclusion is false. A negative free logician might not be troubled by this result. In fact, someone who takes all universally quantified sentences to be true on the empty domain might not be troubled by this result either. However, on NgFL there are populated-domain counterexamples to the argument (!) and no one should admit those without good reason. The seeming validity of the argument requires NgFL to at least justify its loss.

Three Arguments for Negative Free Logic

The first argument comes from Burge. He gives the following reductio argument against classical logic with identity.¹⁷¹ Suppose that ‘Pegasus’ is a non-referring singular term.

1. $\forall x x = x$
2. $\therefore Pegasus = Pegasus$
3. $\therefore \exists y y = Pegasus$

Universal Instantiation justifies the inference from 1 to 2, and 3 follows from 2 by Existential Generalization. Since 3 is unmistakably false, Burge says that “unless we regress to Russell or Frege, we must either alter identity theory or restrict the operations of instantiation and generalization.”¹⁷² For Burge, if ‘Pegasus’ does not refer then ‘Pegasus = Pegasus’ is not about anything and so cannot be true (and so cannot follow from 1). The assumption of bivalence then leads Burge to the conclusion that ‘Pegasus = Pegasus’ and other simple predications containing ‘Pegasus’ are false.¹⁷³ This assumption of bivalence of simple predications merely begs the question against the proponent of a neutral free logic. However, if bivalence could be justified then so could NgFL.

¹⁷¹ Burge, “Truth and Singular Terms,” 311. Scott Lehmann presents a similar argument in “More Free Logic,” 204. However, he does not use it to specifically endorse the rejection of EG.

¹⁷² Burge, “Truth and Singular terms,” 311.

¹⁷³ Ibid., 313.

It seems clear from the argument that Burge's (quoted) claim is not quite right; to escape the negative consequences of his reductio argument, we need not (to avoid 'regressing' or altering identity theory) restrict the operations of instantiation **and** generalization. Disallowing one or the other inference would avoid the absurd conclusion. Here is why Burge is committed to altering both—unlike neutral free logic which saves existential generalization and loses only universal instantiation: if you reject universal instantiation then you must avoid the following work-around of it:

1. $\forall xP(x)$
2. $\sim\exists x\sim P(x)$ *QN*
3. $\sim P(a)$ *assume for RAA*
4. $\exists x\sim P(x)$ *EG*
5. $P(a)$ *by RAA*

The equivalence of $\forall xP$ and $\sim\exists x\sim P$ is valid on NgFL. The only potential trouble case is the empty domain, but on the published NgFL treatments of the empty domain, universal claims are all true and existential claims are all false. So, obviously the equivalence holds for the empty domain. If the domain is non-empty then the proof of the equivalence is easy: whether $P(x)$ contains a non-referring name or not, $P(x)$ is true for every domain element substituted for x if and only if no domain element will make $\sim P(x)$ true. Since there is no third value, $\sim P(x)$ is true for no domain element if and only if $\exists x\sim P(x)$ is false, which is true when and only when $\sim\exists x\sim P(x)$ is true.

If Reductio as Absurdum (RAA) is legitimate then the step from 3 to 4 justified by EG must be disallowed. RAA should be justified for NgFL, given its commitment to bivalence. Furthermore, as we have seen, Burge's argument for negative free logic relies on reductio ad

absurdum in the meta-language. Burge then would seem to have no desire to abandon RAA in the object language. So, Burge's reason for sacrificing universal instantiation and existential generalization boils down to bivalence.

The second argument is from Sainsbury.¹⁷⁴ Where Burge's argument for negative free logic takes the form of a reductio ad absurdum argument, Sainsbury prefers negative free logic because it saves reductio ad absurdum. Like Burge's argument, it relies on the assumption of bivalence. NgFL, Sainsbury claims, does justice to the commonplace but important practice of using RAA to demonstrate the non-existence of something. A hypothesis that a thing exists can be refuted by observations contrary to the implications of the assumed existence claim. For example, Vulcan was shown not to exist by observations that were inconsistent with its existence. A neutral free logic, on the other hand, cannot admit that 'Vulcan exists' is false, and so could not allow an argument that establishes that it is false. As McKinsey points out, supposing that to refute a claim is to show it false (rather than to show it untrue) is simply a reliance on bivalence.¹⁷⁵ On a neutral free logic $\exists x(x = vulcan)$ could not be proved false, because it could not be false. However, a reductio argument could be employed to establish its untruth. So, if refutation is establishment of untruth, then Sainsbury's argument has no force against the neutral free logician.

The third argument is drawn from Sainsbury's two points in the quoted passage at the beginning of this chapter.¹⁷⁶ It follows from those two points that if a simple predication fails to refer to something then it is false. From (i) it follows that if a simple predication fails to refer to anything then it is not true. From (ii) it follows that the non-referring simple predication is false.

¹⁷⁴ Sainsbury, *Reference without Referents*, 68.

¹⁷⁵ In his manuscript *Consequences of Reference Failure* (p.12)

¹⁷⁶ "Negative free logic is motivated by two thoughts: (i) a true simple predication refers to something and predicates a property which that object possesses (the idea extends to any n -ary simple sentence); (ii) falsehood is failure of truth." Sainsbury, *Reference without Referents*, 66.

Since (ii) is plainly inaccurate if taken to mean that anything that fails to be true is false (consider interjections, questions, planets etc.), it must be restricted. One obvious and charitable way of qualifying the premise is to understand it as ‘any simple predication that is not true is false,’ which is obviously just the expression of bivalence for simple predications.

Now I think “it is there” is a simple predication that is obviously neither true nor false in the absence of a speaker’s denotation assignments to “it” and “there.’ I think that the negative free logician should disagree on this point or else she will be committed to a referent requirement for falsity, but I might be mistaken. In any case, perhaps it is more charitable to interpret the negative free logician as relying on bivalence for *interpreted* simple predications. However one restricts the class of entities for which bivalence holds, there must be *some* restriction, and that seems to show that it is not intuitively absurd to deny bivalence for some class of entities (unless it is the class of entities for which bivalence holds!).

All agree that there is some category of things (declarative sentences, propositions, etc..) such that anything in that category that is not true is false. The category is, roughly, the group of truth-bearers. For negative free logicians, all (interpreted) simple predications are in the category. For neutral free logicians, successful reference is necessary for membership. It is important to keep in mind that a view that “rejects bivalence” is not simply for that reason absurd; every view rejects bivalence for some group containing non-truth-bearers (and every view accepts bivalence for some restricted category of truth-bearers). If a view is to be reduced to absurdity because it rejects bivalence it must be because it is absurd to draw the category boundaries where the view draws them. However, the contents of the category of truth bearers are probably not so obvious that claiming an opponent’s drawing of the boundaries is in-itself absurd—at least not as absurd as rejection of the three seemingly valid arguments lost by NgFL

mentioned above.

The arguments in defense of NgFL all rely on bivalence, so we must now consider the arguments for bivalence.

Bivalence

The principle of Bivalence is roughly that truth-bearers are all either true or false. There are several notable variations. A strong Bivalence principle adds the requirement that no truth-bearers are both true and false.¹⁷⁷ Instead of including falsity the principle can be given in terms of negation: every truth-bearer is either true or has a true negation.¹⁷⁸ Also, the principle can be formulated to apply to a variety of truth-bearers. One might give a version of bivalence for propositions, for statements, for declarative sentences in context, or for assertoric utterances. None of these variations should be confused with the Law of the Excluded Middle (LEM), which holds in any logical system in which $P \vee \sim P$ is a theorem. Bivalence is a semantic principle, and LEM is a syntactic one. The former is about truth and the latter is about derivability.

This is the currently fashionable way to distinguish between the semantic and the syntactic points. Professor Powers pointed out to me in conversation that historically, ‘the Law of the Excluded Middle’ has been discussed as a semantic principle, not a principle about the derivability of the tautology. Only recently, has the phrase ‘Law of Excluded Middle’ come to be used in a sense that differs from bivalence. For instance, in Book IV (Γ) Chapter 7 of the *Metaphysics*—the chapter called “The law of excluded middle defended” in W.D. Ross’s translation—Aristotle says “[T]here cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate. This is clear in the first place if we

¹⁷⁷For the distinction between strong and weak bivalence see Miroslava Andjelkovic and Timothy Williamson, “Truth, Falsity, and Borderline Cases,” *Philosophical Topics* 28, no. 1 (2000): 211-244..

¹⁷⁸For this version of the principle see David DeVidi and Graham Solomon, “On confusions about bivalence and excluded middle,” *Dialogue* 38, no. 4 (1999): 785-799.

define what the true and the false are...[H]e who says of anything that it is, or that it is not, will say either what is true or what is false.”¹⁷⁹ This is clearly a statement of what is now called ‘bivalence.’ Historically accurate or not, I will use the contemporary terminology in discussing the distinction.

Williamson’s Argument

In *Vagueness*, Williamson gives a reductio argument for a version of Bivalence:

(B) If an utterance U says that P, then U is either true or false.¹⁸⁰

He relies on the following two principles;

(T) If U says that P then U is true iff P.

(F) If U says that P then U is false iff \sim P.

Suppose that there is a counterexample to (B): an utterance U that says that P but that is neither true nor false. By (T) and the supposition we get \sim P. By (F) and the supposition we get $\sim\sim$ P. Conjoining the results gives \sim P \cdot $\sim\sim$ P, so there could be no counterexample contrary to supposition.

There are two mildly compelling reasons to present the argument differently: Direct proofs are generally more instructive than indirect proofs; also, an opponent of Williamson’s might claim that his reductio proof merely shows that (B) is not false, not that it is true. One need not agree that the non-falsity of (B) implies its truth unless she already accepted (B).¹⁸¹ In deference to those reasons, here is a direct, conditional-proof (CP) version of the argument:

- (1) Suppose (for CP) that U says that P.
- (2) From (T) and Modus Ponens, it follows that U is true iff P

¹⁷⁹ Aristotle, *Metaphysics*, BK.IV: Ch. 6 in *The Basic Works of Aristotle*, ed. McKeon, Richard (New York: Modern Library, 2001).

¹⁸⁰ Williamson, *Vagueness*, 188.

¹⁸¹ See *Ibid.*, 193.

- (3) From (F) and Modus Ponens, it follows that U is false iff $\sim P$
- (4) From LEM and substitutivity of biconditional it follows that U is true \vee U is false.
- (5) So, U is either true or false.

Closing the conditional proof, and universally generalizing gives the (intended) quantified version of (B). Alternately, step (4) could be accomplished by LEM, Constructive Dilemma and one-directional conditionals in (2) and (3).

Since Williamson's argument uses *reductio ad absurdum*, he relies on LEM. Furthermore since his indirect proof shows that a denial of bivalence entails a denial of LEM, he is giving an argument (that relies on LEM) from LEM to bivalence.

The argument, in either version, relies on three components: LEM, (T) and (F), and either RAA or CP. The neutral free logician rejects all three, but some criticisms are better than others.

Reference Failure and Saying

Williamson specifically rules out using the argument to show that utterances containing non-referring names are either true or false, since in such a case the utterance does not say that something is the case.¹⁸² His interest is in utterances containing vague predicates. He considers a parallel escape for his opponent: perhaps in (borderline cases) of utterances containing vague predicates the utterance says nothing. His response to that challenge is instructive for determining exactly what he intends by 'says.' He says there are three reasons for thinking that borderline cases like 'TW is thin' say something: (a) we understand all the components of the utterance and how they are combined. (b) We know circumstances under which the utterance would be true and false (while saying what it says). (c) More complex utterances containing it

¹⁸²Williamson, *Vagueness*, 196.

(the utterance 'TW is thin') can have truth value, so must say something.¹⁸³

Williamson says that he is not arguing for bivalence for propositions, since an opponent of his could claim that vague utterances express no proposition.¹⁸⁴ Williamson and Andjelkovic defend a version of bivalence for sentences in context (rather than utterances). They define 'Say(s,c,P)' "in terms of propositions[: sentence] s [uttered in context] c expresses the proposition that P."¹⁸⁵ That interpretation offers this reading of bivalence: If a sentence in context c expresses the proposition that P then either s in c is true or s in c is false. They also claim that for their purposes there is no significant difference between considering utterances and sentences in context.¹⁸⁶ All of this together seems to imply that Williamson (in 2000) is arguing for the principle (B2 below) that he said (in 1994) misses the point (for Vagueness).¹⁸⁷

It is possible to reconcile the seeming inconsistency. Williamson might hold that an utterance saying that P is implied by its expressing the proposition that P. So one could deny the antecedent of,

(B2) If an utterance U expresses the proposition that P, then U is either true or false without denying the antecedent of (B), but not vice versa. He might also hold that the antecedents are equivalent, but that defending both (B) and that vague utterances satisfy its antecedent is easier than defending both (B2) and that vague utterances satisfy its antecedent.

Arguably, utterances containing non-referring names do not satisfy the antecedent of (B2). Consequently, the neutral free logician who rejects gappy (or incomplete) propositions is free to accept (B2). However, if contrary to Williamson, one maintains that utterances with non-

¹⁸³Ibid.

¹⁸⁴Ibid., 187.

¹⁸⁵Williamson and Andjelkovic, "Truth, falsity, and borderline cases," 6.

¹⁸⁶Ibid., 42 (See footnote three.)

¹⁸⁷Also, in "Reply to McGee and McLaughlin," *Linguistics and Philosophy* 27 (2004): 113. Williamson presents (T) this way: If an utterance U means that P then U is true iff P. He then says "one could gloss 'means' in (T) [...] as 'expresses the proposition.'"

referring terms *say that something is the case*, then she cannot both accept (B) and endorse NFL. If utterances with empty names do not say anything, then obviously Williamson's argument presents no challenge to NFL. Why might we suppose that Williamson's argument is a challenge to NFL? The answer is not likely to come from the sufficient conditions for saying that Williamson gives ((a) – (c) above).

On a direct reference theory of the sort that NFL is employed to defend, it would not be appropriate to claim that we understand the components of an utterance like “Praxedes was martyred” if the name “Praxedes” does not refer. If the sole contribution of a name is its referent and it fails to provide that, it cannot be that a name is understood in the sense of having its meaning grasped. As Williamson presents (b) the condition seems circular; U must (actually) say something in order for there to be imaginable circumstances in which it says *what it actually says* (and is true). However, the condition might be repaired by appending “if anything” to it. Without imagining a scenario in which “Pegasus” has a referent—thereby imagining a scenario in which what is said by an utterance containing the name (if anything is said) has changed—we cannot imagine a scenario in which “Pegasus has wings” is true or false. Regarding (c), it might seem promising to offer something similar to “if Pegasus can chew then Pegasus has a mouth,” and further claim that the conditional says something (because it is true). However, many direct reference theorists will be unpersuaded by the conditional if they were unmoved in the first place to accept that “Pegasus has a mouth” says something. Some, supervaluationists for example, would accept that a tautology containing an empty name says something (because it is supertrue), but they are not likely to endorse the principle needed: if a complex utterance says something then so does its components.

One could argue that the following utterances say that something is the case despite

containing a token without linguistic meaning or referent:

- (6) X is less than or equal to 0.
- (7) That is an echidna.
- (8) It is raining.

Suppose that (6) is an open sentence ('X' is an unquantified variable) and that (7) is uttered in a context insufficient to supply 'that' with a denotation. If (6) and (7) say something then the conditions (a), (b) and (c) are not necessary. Of course, (6) and (7) will not to Williamson be convincing examples of saying without referring, because the lack of referent is even more obvious than in "Praxedes was martyred." The point of appealing to a non-standard semantics like NFL or supervaluations is that often whether the names refer (or whether the predicates pick out determinate properties) is not obvious. If the requirements for saying are stringent then the result (B) is of limited utility; we would have to settle the issue of whether an utterance says something before settling on a logic. If the requirements are lax then the result is unlikely; if too many utterances count as sayings then some will be neither true nor false. So, one possibility is to argue that the lax version of (T) is false and that the stringent version is trivial. This leaves open the possibility that a true and useful version of the principle lies between the lax and stringent versions. So perhaps a more promising possibility is to just *suppose* that declarative utterances with empty names (and imprecise predicates) say something and criticize Williamson's argument; if that supposition is false then the argument is no criticism at all of non-bivalent free semantics.¹⁸⁸

Objections to Williamson's Argument

There are four families of response to Williamson's argument: One can (a) accept the

¹⁸⁸ Just a reminder: Williamson would agree that his argument is not a defense of bivalence for utterances containing non-referring names. I have just tried to co-opt Williamson's argument for the benefit of NgFL, whose adherents offer no support for the bivalence principle as applied to utterances containing empty names.

conclusion but deny that it is an expression of a substantive or interesting principle of bivalence, (b) reject the reasoning (MP or CP), (c) object to (T) or (F), or (d) give up LEM. Vann McGhee and Brian McLaughlin (M&M) opt for the first of these. Really, M&M argue against Williamson with a dilemma: Either (T) is contingent or it is analytic. If it is contingent then it can be denied without incoherence and if it is analytic then it makes no substantive claim about language. In fact, M&M accept the conclusion (B) but say “it by no means contradicts the thesis that there is semantic indeterminacy” and call it “an innocuous platitude that tell[s] us nothing about meaning fixation.”¹⁸⁹ Williamson responds that analytic truths can yield substantive insight. He cites Leibnitz’s law as an example.¹⁹⁰

Whether or not it succeeds at accomplishing McGhee and McLaughlin’s goals, their criticism of (B) is not helpful in defense of NFL. If statements containing empty names are saying utterances, and a logical system for (the first order fragment of) natural language should illuminate the valid and invalid arguments whose components are saying utterances, then (B) is enough to recommend a bivalent semantics.

Francis Pelletier and Robert Stainton (P&S) have also argued for the first option, albeit differently than M&M. They claim that bivalence is the principle that ‘there are only two truth-values, T and F, and that each sentence has exactly one of these values.’ They say that if his argument is a success, Williamson shows that it is absurd to deny: ‘for each sentence, either it or its negation is T.’ Since it is possible to accept Williamson’s bivalence without accepting *real* bivalence (suppose negation is *exclusion negation* and takes neutral to T; there will be three truth-values but no counterexamples to Williamson’s bivalence), his conclusion is no threat to

¹⁸⁹ McGee and McLaughlin, “Review of Vagueness,” 227-228.

¹⁹⁰ Williamson, “Reply to McGee and McLaughlin,” 115.

bivalence.¹⁹¹

It is not clear that the limited conclusion P&S claim for Williamson is all that his argument demonstrates. However, even if they are right about his conclusion, it would be inconsistent with NFL and endorsement of their response requires sacrificing the intuitive and commonly accepted view that falsity is truth of the negation. NFL rejects exclusion negation; in fact, I argue in chapter four (in the section: Relations and Logical Symbols) that there is no object-level negation in English that takes neutral to true. So P&S have offered a criticism that, even if successful for their purposes, will not aid those of NFL.

Supervaluational semantics also provides a criticism of Williamson's argument. It is obvious that a view that takes supertruth to be truth rejects bivalence—since some sentences are neither supertrue nor superfalse—but does not reject LEM since $P \vee \sim P$ is supertrue on every interpretation. So, supervaluationists must find fault with Williamson's argument; in fact, they find several. For one thing, the necessary truth of (T) is inconsistent with SVS. Suppose that U says that P but that 'P' is not a classical tautology and contains an empty name. Then 'U is true' (if truth is supertruth) will be false on all completions. However, some completions will make P true so that the biconditional in the consequent of (T) will be false on some completions that make the antecedent true (unless contrary to the supposition of this section, the antecedent cannot be true when P contains an empty name). It is also true that SVS rejects RAA and CP. To see that CP is invalid, consider that any model with P (true on all completions) will have T(P), but if P contains an empty term then $P \rightarrow T(P)$ will have a false consequent on all completions and a true antecedent on some completions (so will be neither supertrue nor superfalse. The similar argument that SVS must reject RAA can be seen by assuming the negation of $P \rightarrow T(P)$

¹⁹¹ Francis Pelletier and Robert Stainton, "On 'the Denial of Bivalence is Absurd,'" *Australasian Journal of Philosophy* 81 (2003): 372. They criticize an earlier version of Williamson's argument, but their construals of bivalence could be easily updated from talk of sentences to a principle regarding utterances that say something.

and deriving a contradiction. This is possible despite the fact that just demonstrated that the conditional might be neutral on an interpretation. These objections to the argument are dependent on the machinery of SVS, and because of the flaws in that system, we cannot rely on it for criticisms of (T).

Since neutral free logic rejects Conditional Proof, (T), and LEM, there seem to be many options for criticism. However, for the proponent of NFL, one of the alternatives for criticism is more fundamental than the others. Since Conditional Proof is repudiated for the same reason that (T) fails (and because there might be other equivalent ways of formulating an argument from LEM to bivalence that relies on (T) but not Conditional Proof—for example, one that relies, as does Williamson’s original argument, on substitution of equivalents), I will ignore that avenue of criticism. Given the failure of SVS, there are few remaining options for accepting LEM and rejecting bivalence (even if (T) is not a necessary truth).¹⁹² I will argue that one really ought to reject LEM *if one aims to deny bivalence*. Then I will argue that we *should* reject LEM contra both Williamson and SVS. First, the argument for the importance of LEM:

Another Defense of Bivalence: Argument from the T—Schema

The rejection of (T) by those who reject bivalence is related to their rejection of the T—Schema. An argument similar to Williamson’s (but without the baggage of ‘says’) relies only on the T-schema, the definition of falsity and LEM.

$$(TS) \quad T(P) \leftrightarrow P.$$

If falsity is understood as truth of negation and the equivalence in (TS) implies intersubstitutability, then there is a simple argument from LEM and (TS) to bivalence. (TS)

¹⁹² One argument not mentioned before that LEM implies bivalence is that if you accept the former without the latter then you are incoherent with respect to the probability axioms. It follows directly from the additivity axiom that $\text{Prob}(A \vee \sim A) = \text{Prob}(A) + \text{Prob}(\sim A)$. According to SVS the left-hand side is a logical truth and should equal one, but the right-hand side cannot if A contains an empty term. By contrast, NFL is consistent with accepting all of the axioms (provided that the probability function is defined only for sentences with a truth value (non neutral)).

licenses the replacement of 'P' with 'T(P)' and ' \sim P' with 'T(\sim P).' So, after replacement LEM becomes 'T(P) or T(\sim P),' which is just bivalence given the definition of falsity (as truth of negation).¹⁹³

The obvious first step is to deny the validity of (TS). If P is neither true nor false then (TS) will be neither true nor false. The intuitive appeal of (TS) is accounted for in a familiar way—the same way that explained the appeal of (T) and the same way neutral free logic accounts for it; T(P) co-necessitates P. That is, any interpretation on which P holds is also an interpretation on which T(P) holds and vice versa. The simple argument from LEM no longer goes through because co-necessitation does not license inter-substitutability like co-implication does. Substitution is not licensed because Modus Tollens fails for necessitation.¹⁹⁴ For example, it might be that \sim T(P) but not that \sim P, since \sim T(P) can hold in an interpretation where P does not hold *and* \sim P does not hold.

However, things are not so easy. Neutral free semantics, as I am defending it seems to still have a problem. If we want T(P) to be false when P is neither true nor false (as we obviously should), then the truth predicate ought to applied in the following way,

A	T(A)
T	T
F	F
neither	F

If so, a truth functional three-valued account that uses the weak Kleene tables (a neutral sentence

¹⁹³For an argument like this, see van Fraassen's, "Presupposition, Implication and Self-reference," 212-213. Also see Field, *Truth and the Absence of Fact*, 283,287.

¹⁹⁴Van Fraassen, "Presupposition, Implication and Self-reference," 206-207.

poisons any complex sentence containing it) should allow that the following argument is valid (is a necessitation):

$$\begin{array}{l} \underline{(\dots A \dots)} \\ \therefore (\dots T(A) \dots) \end{array}$$

That is, replacement of A with $T(A)$ is truth preserving. Suppose that a premise containing A is true. Since we have weak Kleene tables, A is not neutral. So, $T(A)$ will have the same truth value as A . Since we are considering a truth functional semantics, the resulting sentence will get the same value as the original.

The substitution is also licensed for a truth functional account with strong Kleene tables since the only neutral components allowed in true, complex sentences are the ones that could be false without affecting the truth of the complexes containing them. So, if the premise is true and A is neutral then the conclusion will remain true even with the false $T(A)$ substituted for the (irrelevant) A .

This substitution is not licensed by SVS since under it the connectives are not truth functional.¹⁹⁵ A supertrue, complex premise might contain a neutral component. Replacing neutral components (of supertruths) with falsehoods could result in either a supertruth or a neutral sentence. For example, suppose A is neither true nor false. $A \vee \sim A$ is supertrue but $T(A) \vee \sim A$ is neither true nor false. Also $(B \vee \sim B \vee A)$ is supertrue, as is $(B \vee \sim B \vee T(A))$.

The point is this: neutral free logic sanctions the replacement of A with $T(A)$ and that is all that is needed for the simple proof of bivalence to go through; For NFL, $P \vee \sim P$ necessitates $T(P) \vee T(\sim P)$. So, the important criticism for the defender of NFL is not the rejection of (TS) but rather the rejection of LEM. Before arguing against LEM, I consider one

¹⁹⁵ The general loss of truth functionality can be easily seen by contrasting sentences like ' $Wp \vee Wp$ ' and ' $Wp \vee \sim Wp$ ' where ' Wp ' is neither true nor false. The example in the text shows the breakdown in truth-functionality that is relevant to the failure of substitutivity at issue here.

more seeming wrinkle for NFL: The argument from the T—Schema can be given as a reductio without obvious reliance on LEM. Suppose that P is neither true nor false, so it is not the case that T(P) and it is not the case that T(\sim P). By substitutability from (TS) it follows that it is not the case that P and it is not the case that \sim P, which is inconsistent.¹⁹⁶

This version of the argument, in dodging one, opens up three possible criticisms for NFL. (a) The argument relies on the kind of substitution that NFL prohibits; NFL does not allow the substitution of T(A) with A because \sim T(A) does not necessitate \sim A (see table above). (b) J. C. Beall argues interestingly that the conclusion is not really a contradiction given an ambiguity in negation between choice and exclusion negation.¹⁹⁷ (c) Reductio arguments secretly rely on LEM since deriving a contradiction from P merely shows that P is not the case. That is only evidence that \sim P is the case if we can be assured that either P or \sim P is the case. So criticisms of LEM are criticisms of this version of the T—schema argument too.

Since, in the face of the strengthened T—schema argument (that relies only on replacement of A with T(A)) there seems to be no alternative for NFL other than the rejection of LEM, that seems to be the right approach in response to Williamson's similar argument.

The Law of Excluded Middle

Neutral free logic rejects LEM (and all other propositional tautologies). That is, according to all versions of NFL, there are no unquantified logical truths. As we will see below, it is important to distinguish between that kind of 'rejection' of LEM and another kind: accepting that on some possible models it is true that $\sim(P \vee \sim P)$. The criticisms below establish only the former sort of rejection of LEM. However, before considering those criticisms of LEM, first consider two defenses of it; the first claims that its denial is contradictory and the second claims

¹⁹⁶ J.C. Beall, 'Deflationism and Gaps,' *Analysis* 62, no.4, (2002): 301

¹⁹⁷ *Ibid.*

that its denial is impractical.

Defenses of LEM

In intuitionist logic, negation is contradiction and truth is demonstrability. As a consequence, intuitionist logic rejects LEM and the classically equivalent Double Negation reduction ($\sim\sim P::P$). Understanding negation and truth as the intuitionist does, LEM implies that every P is either demonstrable or demonstrably contradictory, and Double Negation implies that if it is demonstrable that the contradictoriness of P is contradictory (that the mere consistency of P is demonstrable) then P is true. The philosophy of mathematics (Intuitionism) that suggests intuitionist logic is not one I will defend; I am concerned only with an early criticism of intuitionist logic: that its rejection of LEM makes it contradictory.

Marcel Barzin and Alfred Errera in a 1927 paper present such a criticism.¹⁹⁸ My reconstruction of that argument is drawn from two sources, neither of which give a translation of the French original: Alonzo Church's 1928 reply¹⁹⁹ and Mancosu, Zach, and Badesa's *Development of Mathematical Logic from Russell to Tarski*.²⁰⁰ Barzin and Errera invented an object-level, unary logical connective ($'$) that is understood semantically to mean 'is neither true nor false.' So, if for some P , it is not the case that $P \vee \sim P$ then it will be the case that P' . They argue that as a consequence, the intuitionist must accept a law of the excluded fourth—that is, for all substituends P , $P \vee \sim P \vee P'$ is true. From the definition of $'$ and the principle of excluded fourth they derive a contradiction. Because it is in French, that paper and the derivation it contains is opaque to me, so below, I piece together their argument from the sources mentioned.

¹⁹⁸ Barzin, Marcel and Alfred Errera, "Sur la logique de M. Brouwer," *Acad'emie Royale de Belgique, Bulletin* 13 (1927): 56–71.

¹⁹⁹ Church, Alonzo. "On the law of excluded middle," *Bulletin of the American Mathematical Society* 34, no.1 (1928): 75-78.

²⁰⁰ Paolo Mancosu, Richard Zach and Calixto Badesa, "The development of mathematical logic from Russell to Tarski, 1900-1935," in *The Development of Modern Logic* ed. Leila Haaparanta, (New York: Oxford University Press, 2008) 107.

It is part of Alonzo Church's reply to Barzin and Errera that the definition of P' is contradictory. In a passage that seems to reveal Church's position as well as the thrust of Barzin and Errera's argument, Church says:

It is not possible, as an alternative to the law of excluded middle, to assert that some proposition is neither true nor false, because by so doing not only the law of excluded middle would be denied but also the law of contradiction. In fact, to assert that a proposition p is not true and is also not false is to assert at once $not-p$ and $not-(not-p)$ and consequently to assert that $not-p$ is both true and false.²⁰¹

So here is a possibility for Barzin and Errera's argument: by the principle of excluded fourth (which the denier of LEM must accept) if $P \vee \sim P$ is untrue then P' is true. From the definition of P' , it follows that $\sim P$ and $\sim\sim P$, which is a contradiction. So, $P \vee \sim P$ is not untrue.

Church's response that the definition of P' is contradictory does not seem to be the best response. His other criticism—that the argument relies on LEM—is more to the point, since roughly the same argument can be given without any reference to P' :

Suppose LEM is denied. Then for some substituent P , it is not the case that $P \vee \sim P$. So it must be that for some substituent P , $\sim(P \vee \sim P)$. By deMorgan's theorem, that implies that $\sim P \cdot \sim\sim P$ which is a contradiction.

The important point to make in response (which Church also makes) is that denying $P \vee \sim P$ is not the same as accepting $\sim(P \vee \sim P)$. To suppose that it is, is to insist on LEM! One could conceivably accept neither $(P \vee \sim P)$ nor $\sim(P \vee \sim P)$. This point also seems to undermine Church's quoted claim above. To assert that proposition P is not true and is also not false is **not** (!) to assert at once $\sim P$ and $\sim\sim P$. Denial of $P \vee \sim P$ only results in contradiction if denial is assertion of negation.

Another, very different, defense of LEM is a practical one. It is suggested (but not endorsed) by Hartry Field. Abandonment of LEM in cases of reference failure and

²⁰¹ Church, "On the law of excluded middle," 75.

indeterminacy mean that LEM ought to be abandoned whenever reference failure and indeterminacy is possible. But this is everywhere. Since reference failure and indeterminacy are possible even in meta-theoretical reasoning, classical logic will be unusable for nearly every purpose.²⁰²

This practical worry should not bother us too much, because as a practical matter, defending an instance of LEM or any other classical tautology where it is true and necessary for reasoning will not be challenging. One must only, on the semantics defended in the following chapter, suppose that the constituent names refer in order to assume as a premise an instance of a tautology containing those names. In fact, if it were any easier to establish the truth of an instance of a tautology, it would be possible (by an argument similar to that in the introduction) to erroneously deduce *a priori* the existence of contingent objects! I turn now to criticisms of LEM. The first of those again comes from Field.

Criticisms of LEM

In the most obvious sense, acceptance of LEM is acceptance of a logic that makes every substitution instance of $P \vee \sim P$ a logical truth. Rejecting LEM is then maintaining that the logic that best models natural language does not make every substitution instance of $P \vee \sim P$ a logical truth. Defense of LEM is not a task of derivation, but a task of arguing for a logic on which it is valid. Here is one substantial result from that trivial insight: a defense of LEM that says that a contradiction from its denial shows only that *on the logical system used to derive the contradiction*, LEM cannot be consistently denied. Assuming that that logical system is the appropriate one will just beg the question against those who reject LEM. The inferences used in the derivation will themselves need to be defended.

It follows from the foregoing that a rejection of LEM could be defended by finding

²⁰² Field, *Truth and the Absence of Fact*, 287.

untrue natural language disjunctions that are best symbolized as substitution instances of $P \vee \sim P$. The first two criticisms here are in that style. The third takes the form of a meta-linguistic argument.

Field's Criticism

Hartry Field proposes that there is no fact of the matter about *exactly* when Jerry Falwell's life began. Conception, Birth and all events between have duration, so even if it is a fact that JF's life began at one of those events it still seems absurd to say that there was an exact nanosecond during that event at which his life began. However, we can derive that there was such a time from LEM and a few uncontroversial premises.

- (9) $(\text{JF's life had begun by } N) \vee \sim(\text{JF's life had begun by } N)$
- (10) JF's life had not begun by time 0 and had begun by time 10^{18}
- (11) For all times N and M, if $N < M$ and JF's life had begun by N then it had begun by M.

From finitely many instances of (9)—all instances of LEM—and “a minimal amount of arithmetic and logic” it follows that

- (12) There is a unique N_0 such that JF's life had begun by N_0 and not by N_0-1 .

This argument is nearly verbatim from Field's paper “No Fact of the Matter.”²⁰³ Since (12) is plainly false, at least one of the required instances of (9) is untrue. The argument which, as I have presented it, is a *reductio ad absurdum*, relies on LEM. This however, is no problem for the argument, since it is a *reductio* of LEM.

If successful, Field's argument does suggest that there are natural language instances of LEM that are neither true nor false; however, it does not highlight the sort of exceptions that neutral free logic respects. In fact, on the semantics endorsed in chapter four, every model wherein ‘JF’ refers is a model in which every instance of the schema (9) is true. I offer Field's

²⁰³ Field, Hartry, “No Fact of the Matter,” 458.

argument as a way to soften up the reader. The next objections are closer to capturing the violations of LEM that NFL tolerates.

True of nothing

‘Sherlock Holmes wore size ten shoes or he did not.’ This sentence is seemingly untrue since there is no fact that would make either the left or right disjunct true. The negative free logician will accept the right disjunct, saying that it is not the case that Holmes wears size ten shoes since it is not true that he does. An intuition to the contrary might be explained by appealing to a small scope negation interpretation: SH wore size ten shoes or SH wore non-size ten shoes. The negative free logician admits that that disjunction seems not to be true, but, she says, it is not best symbolized as an instance of LEM. NgFL cannot get off the hook so easily, however.

Consider another instance of LEM:

$$\star \quad (\text{SH has 0 friends}) \vee \sim (\text{SH has 0 friends})$$

Suppose that the name ‘SH’ is empty so that \star is neutral on NFL. The negative free logician must accept one of the disjuncts. If she accepts the right disjunct then she loses the evident connection between quantifiers and numbers defended in chapter one. This is so since ‘SH has no friends,’ which comes out true on NgFL when ‘SH’ is empty (‘ $\exists x \text{ Friends}(x, \text{SH})$ ’ is false because all substitutions for x come out false.), would not imply that ‘SH has 0 friends’. If the negative free logician instead accepts the left disjunct of \star , then she suffers the embarrassing loss of the obviously valid argument:

SH has 0 friends.

Joe has 1 friend.

\therefore Joe has more friends than SH.

On NgFL, the argument is invalid if, according to supposition, premise one is equivalent with ‘SH has no friends.’ The conclusion seems to be an atomic binary relation whose constituents are SH and Joe, so it is false for NgFL if ‘SH’ is empty.

One possible response is to claim that the conclusion quantifies over numbers and somehow makes reference to the sets *friends of Joe* and *friends of SH* (e.g., $\exists x \exists y$ s.t. x is the number of Joe’s friends and y is the number of SH’s friends and $x > y$). In that case, since ‘SH’ and ‘Joe’ would not appear as names in the conclusion, but rather as parts of predicates or sets, either (1) the argument is invalid, (2) the premises are really paraphrases of awkward sentences that do not contain names at all (e.g., ‘0 is the number of *friends of SH*’), or (3) some additional premises would be necessary (e.g., if SH has 0 friends then the number of *friends of SH* is 0). Only the third option is at all plausible. However, it has the unfortunate drawback that adding more premises to the argument (e.g., ‘Ann has 0 friends’ and Barb has 0 friends’) requires that the argument rely on even more implicit premises—one for each name appearing in the premises.

Because the conclusion cannot be stated with the standard quantifiers, NFL also must admit that any such argument is enthymematic. However, validity is preserved, no matter the number of premises, by reliance on just a single additional premise:²⁰⁴

★★ For all a, b, x, y if a has x friends and b has y friends and $x > y$ then More friends(a, b).

For NgFL (on the assumption that premise one is equivalent with ‘SH has no friends’), there are counterexamples to the validity of the argument on models where the sentence ★★ is true!

Another possible response to the loss of the above argument is to claim that the

²⁰⁴ The same (or similar) premise is required in order to make the argument classically valid.

conclusion is an infinite disjunction of all the ‘more-than’ possibilities (e.g., (SH has 0 friends and Joe has one) \vee (SH has 0 friends and Joe has two) \vee ... (SH has 17 friends and Joe has 29) \vee ...). This saves the validity of the argument but only at the cost of an enormous mismatch between surface structure and logical structure.

The best response is to admit that there are some instances of LEM without truth value.

Metalinguistic argument from empty domain

For another argument that gets at the heart of the matter as far as NFL is concerned, consider the following two sentences.

- ★ Something is such that it either has friends or it does not.
- ★★ Sherlock is such that either he has friends or he does not.

Their most reasonable symbolizations:

- $\exists x(Fx \vee \sim Fx)$
- $Fs \vee \sim Fs$

My argument:

Sentence ★ is possibly untrue.

So, sentence ★★ is possibly untrue.

Were there no objects at all (on the empty domain), the existentially quantified ★ would not be true. If it is untrue that something is such that X then it cannot be true that Sherlock is such that X. So, the argument is plainly valid, the premise is hardly controversial (absent a proof of the necessity of beings), and the conclusion is inconsistent with LEM.

A critic can have only three options: reject the premise, reject the validity of the argument, or claim that the conclusion is consistent with LEM. For the first option, there are two possibilities: claim ★ would be true in spite of the possibility that there be nothing at all or claim

that there is no possibility that there be nothing at all. The first of these is absurd and the second is in need of an argument.²⁰⁵

The only method of reconciling the conclusion with LEM is by rejecting the symbolization of $\star\star$ as $\circ\circ$. This might be plausible if the negation took small scope in the English sentence (e.g., SH is footed or footless), however the metalinguistic argument clearly gains no benefit from reliance on intuitions about that interpretation of $\star\star$.

A defender of LEM, at least a negative free logic defender of LEM, might say that the argument is just a contraposition of an illegitimate existential generalization. The argument is just a disguised EG from $\circ\circ$ to \circ . That is obviously an impermissible inference from the standpoint of negative free logic, so, NgFL says, the metalinguistic argument is invalid.

On the contrary, you would have to be strongly committed to NgFL for reasons independent of LEM and bivalence (to avoid circularity) to see the argument as invalid. However, as we have seen, reasons independent of LEM and bivalence for accepting NgFL are elusive.

Conclusion

If the foregoing is correct then there are no great reasons to insist on either bivalence or LEM; in fact there might be some good reasons not to accept them. Consequently, the support for negative free logic vanishes, and in light of the argument in the introduction, the only remaining alternative is a neutral free logic. It remains to elucidate the specifics of that semantics. I turn to that project in the final chapter.

²⁰⁵ I remember a talk at Wayne state University by Peter van Inwagen in which he argued that necessarily there is some object (but not that some object exists necessarily). His argument relied dubiously on the Principle of Sufficient Reason, but beyond that I cannot recall (or find) any of it. Also, in conversation Larry Powers attributed the following argument to Alvin Plantinga: if there were nothing there would be the proposition that there is nothing. So there must be something.

CHAPTER FOUR: SEMANTICS FOR NEUTRAL FREE LOGIC

Choices for a Neutral Free Semantics

Once we have decided on a Neutral Free Semantics, several decisions must be made. Many of the options below appear as questions in Lehmann's "More Free Logic" but some do not.²⁰⁶ I present the questions then argue for my answers to them. Then I present the corresponding semantic account. After that I consider one troublesome objection to that semantic account: the intuitive appeal of existentially hedged sentences.

The Domain

In classical logic domains are required to have at least one element. Many classical logical truths (CLTs) are lost when the empty domain is permitted. One such class of CLTs are the non-vacuously, existentially-quantified CLTs. After deciding on an *inclusive* or *universally free* logic—one that permits the empty domain—it remains to decide how the truth values of sentences are determined on the empty interpretation. One standard approach is to treat all universally quantified sentences (even vacuously quantified sentences) as true and all existentially quantified sentences as false (call this the standard inclusive logic). Another, non-standard approach is to treat all non-vacuously, universally quantified sentences as true and non-vacuously, existentially quantified sentences as false, and to treat vacuous quantification as idle (call this *idle-quantifier* inclusive logic). Both approaches have two serious disadvantages: (1) since no functions map to the empty set, the semantics cannot make use of either a denotation or an assignment function. Also (2), these approaches invalidate many of the intuitively valid 'rules of passage'.^{207 208}

²⁰⁶ Lehmann, "More free logic," 219.

²⁰⁷ The rules of passage, useful for moving quantifiers to the beginning of a formula (prenexing), appear, though not by that name, in W.V.O. Quine's *Methods of Logic* (New York: Henry Holt and Co., 1959), 168.

For example, the following argument is intuitively valid:

Everyone, despite the fact that some people are unselfish, is self-interested.

So, some people are unselfish and everyone is self-interested.

Limiting the domain to people, the argument is most naturally rendered:

$\forall x((\exists yUy) \cdot Sx)$

$\therefore \exists yUy \cdot \forall xSx$

Since the first premise is non-vacuously quantified, on both the standard and idle-quantifier inclusive logic, it is true on the empty interpretation. The conclusion, since it contains as a conjunct, the non-vacuously quantified ‘some are unselfish,’ is false on both inclusive logics. This lost argument is worse than the typical loss a neutral free logician is prepared to take on at least two (related) counts. It is a classically valid argument that could have true premises and a *false* conclusion, not merely a neutral conclusion. Also, the counterexample does not arise from the possibility of an empty name; the conclusion contains no symbols not already in the premises.

Also, other intuitively valid arguments are lost by both of the inclusive logics mentioned so far. For example, on the empty domain, the following (obviously valid) argument has a true premise and an untrue conclusion (on standard *or* idle-quantifier inclusive logic):²⁰⁹

Anyone identical to Samuel Clemens is an author.

So Samuel Clemens is an author.

One more argument against both approaches: If quantified sentences have truth value on the empty domain then either vacuous universal quantification is true on the empty domain

²⁰⁸ All of the following are invalidated by either approach: $\forall x(A \cdot Bx) \leftrightarrow (A \cdot \forall xBx)$, $\exists x(A \vee Bx) \leftrightarrow (A \vee \exists xBx)$, $\exists x(A \rightarrow Bx) \leftrightarrow (A \rightarrow \exists xBx)$, $\exists x(Ax \rightarrow B) \leftrightarrow (\forall xAx \rightarrow B)$.

²⁰⁹ McKinsey’s inclusive semantics avoids loss of this argument. His quantifier rules include a clause that neutralizes *any* sentence an empty name. Since all names are empty on the empty domain, it cannot provide a counterexample to the validity of the argument.

(standard) or vacuous quantification is eliminable (idle-quantifier). If universal quantification is always true on the empty domain then $\forall xP/\cdot P$ is invalid. If it is eliminable then $\sim P/\cdot \exists x\sim P$ is valid unless $\forall xP \neq \sim \exists x\sim P$. Both results are absurd, so quantified sentences do not have truth value on the empty domain.

I plan to allow empty domains because ‘something exists’, or $\exists x\exists y(x=y)$, though obviously true if utterable, should not be *logical* truth (logic alone should not guarantee that something exists). Really, I would prefer to require an element in the domain and dispense with the complications of permitting the empty domain, but I have no argument for the *logical* necessity of the existence of something.

To satisfy the motivating requirement that $\exists x\exists y(x=y)$ not be a logical truth, while at the same time preserving the use of denotation and assignment functions and the rules of passage, my semantics merely assigns no truth value in the absence of domain elements. There are no elements for the denotation function to assign to the variables. The effect of this is to make all sentences truth-valueless on the empty interpretation, and consequently is to eliminate ALL logical truths and falsehoods.

The move is justified for atomic sentences because nothing can be truly or falsely said of nothing. This pill will not be a hard swallow for anyone who accepts a neutral free logic. I will not defend it further here. The move is justified for quantified sentences because of the inferences it saves. Not only does it preserve the rules of passage, but it also preserves what is intuitively correct: that vacuous quantification is impotent, so that both $\forall xA$ and $\exists xA$, where A contains no free x , always has the same value as A .

As I see it, the main benefit of this approach is that it satisfies the guiding intuition (that ‘something exists’ should not be a logical truth) while preserving all and only the valid

inferences of a neutral free logic that ignores the empty domain.²¹⁰ Proof: The addition of a single model could not make a previously invalid argument valid (because validity over a set N of models implies validity over any subset of N). To be responsible for the loss of a valid inference, the empty model would have to provide a counterexample: true premises and an untrue conclusion. Of course, if no sentences are true on the empty model then it cannot provide such a counterexample.

There are three seeming drawbacks to lack of truth values on the empty domain:

Drawback one: There would be no logical truths at all. Even the consolation prize of neutral free logic, $\forall x(Px \vee \sim Px)$, fails to be a logical truth on this account. In fact, NFL can claim **no logical truths or logical falsehoods** if there is an interpretation where no sentence is either true or false.

However, as a general rule, the loss of logical truths should not discourage us, since as I see it, the foremost aim of logic is to capture the valid inferences, not to say which sentences are true. As an illustration of just how fickle our intuitions about logical truths are, the consolation prize sentence would already be rejected as a logical truth by someone who allows vague predicates and takes a neutral stance to them. It might seem to be an especially bad violation that on my proposed semantics $\forall x(Px \cdot \sim Px)$ is not logically false. However, at least on my semantics the sentence is a logical untruth. On both standard and idle-quantifier inclusive logic there is a model on which the sentence is true!

Drawback two: No one could truly say ‘nothing exists.’ ‘It is false that something exists’ is intuitively true (not truth-valueless) if the domain is empty.

This objection is ultimately unconvincing, however. Even less important than our intuitions about logical truths are our intuitions about **contingent** truths on an especially peculiar

²¹⁰ It also preserves the standard semantic tool of assignments and x-variants.

model. These should not carry much weight at all. So what is required is an argument that does not merely rely on intuitions about contingent truths. Here's a possible metalinguistic argument for the logical possibility of 'nothing exists.' Since we are allowing the domain to be empty, it is not logically true that something exists.

It is not logically true that something exists.

So, it is logically possible that nothing exists.

This argument should not be persuasive to the neutral free logician since it assumes bivalence. Compare for example: it is not logically true that 'Pegasus exists,' so it is logically contingent that 'Pegasus does not exist.' The fact that something is not true in every model does not guarantee a model in which it is false.

Drawback three: Sainsbury's intuitively invalid inference ' $\forall xFx$ therefore $\exists xFx$ ' remains valid.²¹¹ This intuitive evidence is especially unpersuasive since the only conceivable counterexample to the inference is the empty domain and the intuition that the premise is vacuously true on it. In fact, Quine used the conditional form of the inference as an illustration of a logical truth formed from a **valid** implication.²¹² The inference might seem worse to Sainsbury than it is since it sounds similar to the obviously invalid, 'every F is G so Some F is G.'

Identity

For neutral free logic, choices are limited on identity: non-referring terms are poison. One might try to accommodate the intuition that $\exists x(x = c)$ is false when c does not refer by making identity sentences false when at least one term does not refer.²¹³ Lehmann, who is

²¹¹ Mark Sainsbury, *Logical Forms*, second ed. (Malden, MA: Blackwell, 2001), 247.

²¹² Quine, *Methods of Logic*, xv.

²¹³ Dolf Rami argues for this approach in the unpublished 'Non-Standard Neutral Free Logic, Empty Names and Negative Existentials (Draft Two)'. (2012. <http://philpapers.org/archive/RAMNNE>) His argument: (1) $E!p$ is false if

motivated to accommodate that intuition, does so by modification to the rule for the quantifier. I believe Lehmann's approach has its problems, but it is a minor improvement over the current proposal to modify to the rule for identity. Both approaches lose unrestricted existential generalization. The currently proposed suggestion (as does negative free logic) countenances true instances of $a \neq a$. It might seem bad enough to allow neutral instances of it, but it is worse to allow *true* instances. Also, the obviously valid argument above is no longer valid on a bivalent identity view:

Anyone identical to Samuel Clemens is an author.

So Samuel Clemens is an author.

On a bivalent identity view, the premise is vacuously true if the name 'Samuel Clemens' has no referent, but the conclusion is neither true nor false.

Here is another valid argument (an instance of the prototype argument) lost on the bivalent identity view:

Everything that is not self- identical is strange.

Pegasus is not self-identical.

So, Pegasus is strange.

If the argument has the representation below then there is an interpretation whereupon its premises are true on the bivalent identity view (the first premise is vacuously true) but its conclusion is untrue (when 'Pegasus' is empty).

$$\forall x(x \neq x \rightarrow Sx)$$

$$\underline{p \neq p}$$

$$\therefore Sp$$

'p' is empty. (2) $E!p$ is equivalent to $\exists xPx$. (3) Bivalent quantifiers are unacceptable. (So) Identity must take only classical values [$s = t$ is false iff there is no x such that x is the referent of 's' and x is the referent of 't'].

Since the bivalent identity view has insurmountable problems, the only real alternative—non-referring terms result in lack of truth value—is the best alternative.

Relations and Logical symbols

Empty terms contribute no value to the atomic sentences that contain them. Since the value of a sentence is a function of the values of its parts, atomic sentences containing empty names have no value. Weak Kleene tables are clearly in the same spirit. If the value of a complex sentence is a function of the values of its parts and one of the parts has no value, then the complex will have no value. Following Lehmann, I will sometimes abbreviate this consideration with ‘NINO’ (for ‘no input, no output’).²¹⁴

Sainsbury, in *Reference without Referents*, proposes a general difficulty for this sort of logic. In his words,

[I]t seems hard to exclude the possibility of an operator, say ‘Neg’, which can attach to any intelligible sentence S to form a truth just on condition that S is not true...Even if S is without truth value, ‘Neg S’ is true, and so Frege’s vision, according to which truth valuelessness dominates all embeddings, would falter.²¹⁵

As a matter of conceivable logical machinery, such an operator is easy to define. However, that there is a natural language, object-level operator that corresponds to the operator is much more difficult to defend. If ‘Neg’ is defined in terms of the truth of ‘S’ then it seems more appropriate to say that it is a meta-linguistic operator. So, someone who uttered the natural language equivalent of ‘Neg S’ (‘It is untrue that S’?) would be *mentioning* the sentence S, rather than *using* it in a way that could show that Frege’s vision falters.

²¹⁴ Scott Lehmann, “‘No input, no output’ logic,” in *New Essays in Free Logic: In Honour of Karel Lambert*, by Karel Lambert, Edgar Morscher, and Alexander Hieke, eds. (Norwell, MA: Kluwer, 2001):147-155 .

²¹⁵ Sainsbury, *Reference without Referents*, 67.

The reasons for taking ‘neg’ to be a metalinguistic operator are the similarities between it and quotation. As with quotation, the operator ‘neg’ is opaque to quantification. Quantifying in is obviously not truth preserving:²¹⁶

[Neg] Pegasus is something.

There is something such that [neg] it is something.

Some opaque contexts are object level (Al wonders if Pegasus lives), so this last point is not decisive. There are however some similarities with quotation not shared with object-level opaque contexts which more strongly suggest that ‘neg’ would be a metalinguistic operator.

Quotation can be used to introduce novel words. So could a natural language ‘neg’ operator, since it returns the value ‘true’ when its argument is truth-valueless. It could not be true to say ‘Al wonders whether flarv is tirpy’, but it would be true to say ‘[neg] flarv is tirpy’.

If empty names provide no problems for ‘neg’ then presumably, neither would empty pronouns or demonstratives. Consider for example the joke sentence: If you cannot say something nice then you should at least make it funny. Prefixing the sentence with the neg operator would yield a truth even though the ‘it’ in the consequent is the equivalent of an unbound variable.²¹⁷ This is similar to quotations and not to other object-level opaque contexts. Suppose context supplies no referent to ‘that’ in ‘Al wonders if that died.’ The sentence lacks truth value. By contrast ‘Al said ‘if that died’’ and ‘[neg] if that died’ could still be true.

If I am right that the hypothetical neg operator is a metalinguistic operator and has no object-level, natural language equivalent, then there is no simple, knock down argument against

²¹⁶ This is adapted from McKinsey’s example of the failure of existential generalization on NgFL.

²¹⁷ At least it is unbound if the sentence is symbolized as it sounds: $\sim\exists x(Nx \cdot Sxy) \rightarrow Fx$.

weak Kleene tables.²¹⁸ On the other hand, there is a simple, knock-down argument for them: NINO.

Quantifiers

The first choice about quantifiers is whether to make them three-valued or two-valued. The main consideration recommending two-valued quantifiers (for a neutral free logic) is negative existentials. Intuitively, it is true to say that ‘there is no Pegasus,’ and bivalent quantifiers can make the obvious symbolization of it true.²¹⁹ However, it seems clear that three valued quantifiers are required by NINO. Since NINO has not been sufficient to convince anyone (since Smiley²²⁰) that three-valued quantifiers are better than two, I will provide another argument, one that takes the form of a dilemma: Either bivalent quantifier rules lose obviously valid inferences (like Quantifier Negation, $\forall x = \sim\exists x\sim$) or they are absurd.

For ease, ignore the empty domain case temporarily. The most natural bivalent universal rule is

(B \forall) $\forall xP$ is true if every value of x makes P true, otherwise it is false.

The natural bivalent existential rule is:

(B \exists) $\exists xP$ is true if some value of x makes P true, otherwise it is false.

These are the rules that Lehmann proposes in ‘More Free Logic.’²²¹ The problem with this pair of rules is that $\exists xP$ and $\sim\forall x\sim P$ do not always have the same truth value (nor do $\sim\exists x\sim P$ and $\forall xP$), furthermore the pair ($\exists xP$ and $\sim\forall x\sim P$) does not co-necessitate. When P contains an empty name

²¹⁸ Dolf Rami also gives an argument that *neg* is not negation in English in his unpublished “Non-standard neutral free logic...,” 24.

²¹⁹ Lehmann, “More Free Logic,” 234, and “‘No input, no output’ logic,” 153. Sainsbury sees negative existentials as a major problem for a ‘Fregean’ free logic. See his *Reference without Referents*, 69.

²²⁰ Timothy Smiley, “Sense without denotation,” *Analysis* 20, no. 6 (1960): 126.

²²¹ Lehmann, “More Free Logic,” 234,35.

the negated side is true but the other side is false. One consequence of this problem is that on this account a statement like,

A. Nothing is taller than Joe

is ambiguous between the non-equivalent

B. There is not anything that is taller than Joe

and,

C. Everything is not taller than Joe.

On the contrary, sentence A is obviously not ambiguous. Furthermore, the argument from B to C is obviously valid, but on the bivalent approach under consideration, B can be true while C is false (when 'Joe' is empty). One more related absurdity: B obviously implies that

D. If Joe is a person he is the tallest person.

But on Lehmann's 2002 bivalent quantifier account, B could be true while D is false. (Or on his account 'There is not anything taller than person Joe' does not imply that 'Joe is the tallest person.')

It is possible for a bivalent quantifier approach to save the equivalence of $\exists xP$ and $\sim\forall x\sim P$ (and $\sim\exists x\sim P$ and $\forall xP$), but the cost is too great. The existential rule that preserves the equivalence of $\exists xP$ and $\sim\forall x\sim P$ for $(B\forall)$ is,

$\exists xP$ is false if some value of x makes P false, otherwise it is true.

This delivers an absurd result. $\exists x(x = Pegasus)$ is true if 'Pegasus' is empty. There is a similar problem for the universal rule that preserves the desired equivalence for $(B\exists)$; $\forall x(x = Pegasus)$ comes out true if 'Pegasus' is empty. That is the absurdity that leads Lehmann to abandon in his 2002 'More Free Logic' the Quantifier Negation equivalence that he endorsed earlier in his 1994 'Strict Fregean Free Logic'.

So the best quantifier rules for NFL are trivalent. Before working out the best trivalent quantifier rules, I will address Lehmann's argument in "Strict Fregean free logic" that \exists does not express existence on trivalent quantifier view. In paraphrase, the argument is:²²²

'Pegasus does not exist' is true if 'Pegasus' does not refer.

On trivalent quantifier rules, ' $\sim\exists x(x = Pegasus)$ ' is neutral if 'Pegasus' does not refer.

So, with trivalent quantifier rules ' $\sim\exists x(x = Pegasus)$ ' does not say 'Pegasus does not exist.'

The strict proponent of NINO will reject the first premise. One cannot say anything true of what does not exist! There is nothing of which to say it. The intuition that 'Pegasus does not exist' is true when 'Pegasus' does not refer can be explained in one of two ways. It might be that the existence claim in the antecedent is just *mistaken for* the metalinguistic claim in the consequent. Also it might be that the non-referring term is understood as a definite description. All of the most persuasive (seemingly true) cases of 'X does not exist' are cases where ready descriptions are available. Consider the following two *unpersuasive* cases:

Jeptonelly does not exist.

Joe does not exist.

It is not intuitively obvious that either one could be true if there is not an obvious, associated description for 'Jeptonelly' or 'Joe' (the first because, as far as I know, no one has the name 'Jeptonelly,' and the second because thousands have the name 'Joe'). Suppose that in some context, an utterance of each seems to be true. An explanation of that fact must, supposing that no description is available, include a metalinguistic claim about the name's failure to refer (or else the context of utterance will make it obvious that the name fails to refer).

²²² Lehmann, "Strict Fregean free logic," 328.

So there is no overwhelming reason to accept bivalent quantifiers and there is a good reason to reject them. Consequently, some trivalent rule is correct. Smiley presents the most straightforward three-valued quantifier rules.²²³ In his words:

The value of $\forall xA$ shall be T if the value of A is T for every value of x; it shall be F if the value of A is F for some value of x.

The value of $\exists xA$ shall be T if the value of A is T for at least on value of x; it shall be F if the value of A is F for every value of x.

For each rule, if neither condition is met then no truth value is assigned.²²⁴ These rules preserve, in the strongest sense, the equivalence (they co-necessitate and always have the same value) of $\exists xP$ and $\sim\forall x\sim P$ and of $\sim\exists x\sim P$ and $\forall xP$. They also seem to give the right value to sentences containing empty names, since such a sentence will not have a truth value for any assignment of domain elements to the free variables. However there is a problem with Smiley's rules as far as I am concerned (but perhaps not given his own concerns): Smiley's rules do not extend naturally to include the empty domain case. He explicitly rules it out.²²⁵

Consider the empty domain and the sentence $\forall x(x = Pegasus)$. The condition for the truth of the universal sentence is trivially satisfied on the empty domain. However, a neutral free logician that is inclined to three-valued quantifiers should insist on the neutrality of the sentence. The situation is similar for the sentence: $\exists x(x = Pegasus)$. For a neutral free logician that wants trivalent quantifiers for NINO reasons, the sentence ought always to be neutral; however Smiley's rules make it false on the empty domain.

²²³ Smiley, "Sense without denotation," 126.

²²⁴ These are essentially the quantifier rules of Pryor's "More on hyper-reliability and a priority;" for him, $\forall x\phi$ takes minimum value of ϕ under all assignments and $\exists x\phi$ takes the maximum value of ϕ . Unlike Smiley however, Pryor allows the empty domain and uses strong Kleene tables.

²²⁵ Smiley, "Sense without denotation," 126.

One way of rectifying the rule is to go back to classical rules and carve out an exception for sentences that contain empty names.²²⁶ I have some reasons for reluctance about this approach: one minor problem is that it, like Smiley's approach, cannot be naturally extended to handle vague predicates. My semantics cannot account for them either but, I believe, it could be extended naturally to do so. Another reason is that it seems ad hoc.

Except for the empty domain, the rules I present below are equivalent to both McKinsey's and Smiley's. They are intended to accomplish the following: (1) preserve the strongest equivalence of $\exists xP$ and $\sim\forall x\sim P$ and of $\sim\exists x\sim P$ and $\forall xP$, (2) assign no truth value to any sentence containing an empty name (on any model including the empty domain), (3) preserve all of the classical inferences whose conclusion contains no names not also appearing in the premises, and (4) assign no truth value to sentences on the empty domain.

Logical Properties

Satisfaction of these desiderata requires becoming comfortable with some revisions to the classical logical truths and to classically valid argument schemata.

Logical truths must be true on every interpretation; otherwise there could be logical truths that are not true. Logical non-falsehoods are interesting, but it would be an error similar to the assumption of bivalence to refer to them as logical truths. Similarly, logical implication (validity) must preserve truth, because truth preserving inferences are the ones we care most about in natural language. It is possible to give a weaker definition of 'valid arguments:' those that never have true premises and a false conclusion. On this weaker validity all of the arguments of classical logic are valid on the NFL semantics given here (this does not hold on standard inclusive NFL logic).

²²⁶ This is McKinsey's approach.

There are no logical truths in the strong sense on the semantics presented here. That means that the classical inference rules that permit zero-premise deductions are lost. For example, Reductio ad Absurdum, Conditional Proof, Tautology and Identity are invalid on any neutral free logic with weak Kleene tables. Here I briefly consider RAA and CP.

Suppose Santa knows he is dead. Because everyone who knows something is not dead, and because if something is known then it is true, it follows that Santa is both dead and not dead. So, Santa does not know he is dead.²²⁷ This, of course, could have an untrue conclusion.

This loss of RAA might seem to be a disaster since it seems to capture a fair amount of actual reasoning. There are few points to be made in response.

A reductio-like derivation rule could be made to require, as auxiliary premises, existence claims for all names in the conclusion, or require at least that all names in the conclusion appear in premises not assumed for RAA or CP. This might seem not to capture the reasoning of someone who argues for the nonexistence of something by supposing its existence (she would not rely on an auxiliary existence claim as a premise). Of course, there are no counterexamples to RAA on this semantics for conclusions that contain no empty names. So, Euclid could, for example, still help himself to his reductio argument that there is no greatest prime. Alternatively, RAA could be recast as an argument for the metalinguistic conclusion that the assumption is untrue (rather than false). This does not seem to be contrary to the reasoning that we engage in when using RAA.

Now briefly consider Conditional Proof: suppose A, derive B; conclude $A \rightarrow B$. If either A or B contains an empty name then the conclusion will be truth-valueless despite the derivation of B from A. Conditional proof is the cornerstone rule of natural deduction systems. In fact, it is

²²⁷ This example is a variation of an example in Benson Mates, *Elementary Logic*, (New York: Oxford University Press, 1972) 216. Mates attributes his example to Origen.

possible to maintain that without it, no deduction system could be a natural deduction system. This threatens the hopes of having a natural deduction system for NFL.

One response is given by Francis Pelletier: A natural deduction system could be given for a logical system with only the Scheffer–stroke. So, natural deduction cannot require conditional proof.²²⁸ However, there is more to say than this.

A CP-like rule could be used if it required existence claims as auxiliary premises.²²⁹ For example, ‘If Al is a student and a teacher then Al is a teacher’ follows from CP by supposing the antecedent and deriving the consequent. If as a premise it is assumed that Al exists then there is no problem with the derivation on a neutral free semantics. Unlike with RAA, this does seem consistent with the mental process of CP. The appeal of a natural deduction system is that it seems to capture natural deductive reasoning. So the loss of CP is not that great if the natural pattern of reasoning could reasonably be said to rely on an existence assumption. There might be one particular use of CP that does not rely on an auxiliary existence assumption: where CP is used to derive a hedged sentence like ‘if Santa exists then he is not an elf.’ However, I do not think that this is a devastating criticism either; I respond to intuitions about hedging below.

In addition to the loss of zero-premise deductions, some classically valid arguments (with premises) fail to be valid on NFL. For example, Addition (and the closely related paradoxes of material implication) and Universal Instantiation are invalid on a neutral free semantics with weak Kleene tables.

All of the lost argument forms have a common characteristic: there is an element (a sentence letter, constant, or relation) in the conclusion that does not appear in the premises. When not working within the assumptions of classical logic, introducing new constants or

²²⁸ Francis Jeffrey Pelletier, "A history of natural deduction and elementary logic textbooks," *Logical consequence: Rival approaches* 1 (2000): 106, 115.

²²⁹ Or it could require that all names in the conclusion appear in premises not assumed for RAA or CP.

predicates obviously poses a problem. In natural language a meaningful sentence in possession of truth value can be transformed to a sentence devoid of truth value with the addition of new terminology. To consider a simple example, merely concatenate ‘snow is white,’ ‘or,’ and a gibberish phrase. The result is obviously devoid of truth value (at least it is if the natural language ‘or’ is truth functional). If our logic is to capture reasoning in natural language then we ought to permit the possibility of invalid instances of Addition and other argument forms that similarly have terms and predicates in the conclusion that do not appear in the premises.

Semantics for NFL

L is a language with finitely many variables and finitely many constants. There is a well-ordered set of additional constants with which to expand **L** when necessary. In addition to finitely many predicates of n -places for $n \geq 1$, there is one logical (two-place) predicate ‘=.’ An n -ary predicate is a capital letter followed by n argument places. The recursive formation rules for sentences are standard; the logical operators \sim , \vee , \cdot , \rightarrow , \leftrightarrow , \forall , and \exists form formulas in the standard way by means of parentheses. Sentences are formulas that contain no free variables.

An interpretation **I** of a language **L** is a possibly empty domain **D** and a denotation function **d**. The function **d** takes all predicates in **L** to extensions in **D**. If **D** is empty then **d** assigns empty extensions to all predicate letters. Also, **d** can take constants in **L** to elements of **D**; **d** can be a partial function on the constants of **L**. If **D** is empty then **d** must be undefined for all constants. An extension of **I** is an expansion of the language to include constants not in **L** and a denotation function **d'** which is exactly like **d** except that it assigns domain elements (if there are any) to the new constants. An α -extension of **L** is an expansion of **L** to include the first constant not already in **L**, call it α , and a **d'** just like **d** except that it assigns a domain element (if

any) to α . Note that if \mathbf{D} is empty then like \mathbf{d} , \mathbf{d}' will assign only empty extensions to the predicates and will be undefined for all constants.

For the valuation rules below, let φ and ψ be formulas of \mathbf{L} that either contain no free variables (they are sentences) or contain at most a single free variable, x . Also, let t_n be a constant of \mathbf{L} . Let $\varphi(x/\alpha)$ be the result of replacing all free occurrences of x in φ with α . Where \mathbf{D} is empty, the rules below assign no truth value.

Values under \mathbf{I} for atomic sentences:

- For a sentence $P(t_1 \dots t_n)$ with $n \geq 1$, if all of $\mathbf{d}(t_1), \dots, \mathbf{d}(t_n)$ are defined then $P(t_1, \dots, t_n)$ is true if $\langle \mathbf{d}(t_1), \dots, \mathbf{d}(t_n) \rangle \in \mathbf{d}(P)$, and $P(t_1, \dots, t_n)$ is false if $\langle \mathbf{d}(t_1), \dots, \mathbf{d}(t_n) \rangle \notin \mathbf{d}(P)$. If at least one of $\mathbf{d}(t_1), \dots, \mathbf{d}(t_n)$ is undefined then $P(t_1, \dots, t_n)$ is neither true nor false.
- For a sentence $t_1 = t_2$, if both $\mathbf{d}(t_1)$ and $\mathbf{d}(t_2)$ are defined then $t_1 = t_2$ is true if $\mathbf{d}(t_1) = \mathbf{d}(t_2)$ and $t_1 = t_2$ is false if $\mathbf{d}(t_1) \neq \mathbf{d}(t_2)$. If at least one of $\mathbf{d}(t_1)$ and $\mathbf{d}(t_2)$ is undefined then $t_1 = t_2$ is neither true nor false.

Values under \mathbf{I} for compound sentences:

- If φ has truth value then $\sim\varphi$ is true if φ is false and $\sim\varphi$ is false if φ is true. If φ has no truth value then $\sim\varphi$ is neither true nor false.
- If both φ and ψ have truth value then $\varphi \vee \psi$ is true if either φ or ψ is true and $\varphi \vee \psi$ is false if both φ and ψ are false. If at least one of φ or ψ has no truth value then $\varphi \vee \psi$ is neither true nor false.
- If both φ and ψ have truth value then $\varphi \cdot \psi$ is true if both φ and ψ are true and $\varphi \cdot \psi$ is false if either φ or ψ is false. If at least one of φ or ψ has no truth value then $\varphi \cdot \psi$ is neither true nor false.

- If both φ and ψ have truth value then $\varphi \rightarrow \psi$ is true if either φ is false or ψ is true and $\varphi \rightarrow \psi$ is false if both φ is true and ψ is false. If at least one of φ or ψ has no truth value then $\varphi \rightarrow \psi$ is neither true nor false.
- If both φ and ψ have truth value then $\varphi \leftrightarrow \psi$ is true if φ and ψ agree in truth value and $\varphi \leftrightarrow \psi$ is false if φ and ψ disagree in truth value. If at least one of φ or ψ has no truth value then $\varphi \leftrightarrow \psi$ is neither true nor false.

These rules produce the weak Kleene tables, which are classical except that a sentence without truth value poisons any sentence of which it is part. The table below is arranged to highlight the point:

φ	ψ	$\sim\varphi$	$\varphi \vee \psi$	$\varphi \cdot \psi$	$\varphi \rightarrow \psi$	$\varphi \leftrightarrow \psi$
T	T	F	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	F	T	F
F	F	T	F	F	T	T
T	N	N	N	N	N	N
F	N	N	N	N	N	N
N	N	N	N	N	N	N
N	T	N	N	N	N	N
N	F	N	N	N	N	N

Values under **I** for quantified sentences:

- $\forall x\varphi$ is true if $\varphi(x/\alpha)$ is true under all α -extensions of **I**. $\forall x\varphi$ is false if $\varphi(x/\alpha)$ is false under some α -extension of **I**. $\forall x\varphi$ is neither true nor false if $\varphi(x/\alpha)$ lacks truth value under some (or every) α -extension of **I**.
- $\exists x\varphi$ is true if $\varphi(x/\alpha)$ is true under some α -extension of **I**. $\exists x\varphi$ is false if $\varphi(x/\alpha)$ is false under all α -extensions of **I**. $\exists x\varphi$ is neither true nor false if $\varphi(x/\alpha)$ lacks truth value under some (or every) α -extension of **I**.
- If the constants are all undefined under **d'** (which can only happen if **D** is empty) then the quantified sentences lack truth value.

This method of making truth depend on all α -extensions, which comes from Boolos, Burgess and Jeffery,²³⁰ has advantages over two other common semantic approaches. Its advantage over assignment functions on variables (in addition to a denotation function on terms) is that on it, variables are treated just like constants. This fits naturally with an understanding of pronouns as variables and with my treatment of the empty domain. There is another common method of evaluating variables like names; it replaces free variables with the first constant of the language not contained in the formula under evaluation and considers possible variations of the denotation function which differ only in what they assign to that constant. Unlike that approach, the one in this section does not unnaturally require that the language contain infinitely many constants.²³¹ Instead, it requires only that new constants be available for expansion of the language; which is more like the natural language situation with respect to names.

²³⁰ George S. Boolos, John P. Burgess, and Richard C. Jeffrey, *Computability and Logic*, 4th ed. (New York: Cambridge University Press, 2002), 117.

²³¹ Susan Vineberg suggested to me this advantage of the language extension approach. I am grateful for the suggestion.

Criticism: Hedging

One consideration has been conspicuously absent in the construction of the semantics: hedging. Intuitively, one can hedge against non-existence. For example, ‘Santa is an elf’ is not a statement true of anything if ‘Santa’ does not refer. However, ‘If there is a Santa then he is an elf’ for some *seems* to express a proposition even if ‘Santa’ is non-referring. There is also another motivation for hedging—it might provide a way to avoid the kind of argument that appears in the first chapter. If logical truths are knowable a priori and they imply intuitively contingent existence claims, then those existence claims would seem to be knowable a priori. If what we really know a priori are hedged claims—not ‘Santa is either an elf or not’ but rather ‘If there is a Santa he is either an elf or not’—then the problem of contingent a priori existence claims vanishes. So can the hedging intuition be accommodated and the problem of contingent a priori existence claims avoided so easily?

Crafting a semantics to account for the intuition is tricky. In this final section of the chapter I will try to say why the most complete neutral free logic account of hedging (Pryor’s 2006b account from “More on hyper-reliability and a priority”) fails, and I will also try to say why any account is likely to fail. That is, I will argue that we should explain away the hedging intuition rather than try to accommodate it.

Following Pryor I will define hedging in terms of semantic power. “An unhedged predication of G to α is one that entails α exists; a hedged predication is one that does not.”²³²

General problems

Hedging, whenever it is attempted, sounds like a conditional sentence. So the most natural logical interpretation of it ought to be (or contain) a conditional; but what kind of conditional? If you want *a priori* knowable claims to be hedged then you want them to be true

²³² Pryor, “More on hyper-reliability and a priority,” 7.

both when the antecedent is not true and when the antecedent is true. This requirement is well-met by a material conditional with a logically true consequent, however, a neutral free logic with weak Kleene tables will make neutral all hedged claims with untrue antecedents. So, perhaps strong Kleene tables in a logic where untrue existence claims are false solve the problem. If the antecedent is not met the hedged conditional is true. This material conditional account (and any material conditional account) has the flaw that it cannot accommodate the intuitive support for half-hedging or vacuous hedging. Consider ‘if Santa exists then he is identical to Pegasus’ and ‘if Santa exists then Pegasus is tethered at the stables of Camelot. Both sentences intuitively imply the existence of Pegasus, but would be true whether or not Pegasus existed as long as Santa did not.

If the material conditional is too weak to serve as a hedging conditional then no conditional that meets a few minimal requirements is sufficient as a hedging conditional. The conditions are:

1. The hedging conditional is true if the antecedent strictly implies the consequent, even if the antecedent contains non-referring terms.
2. Exportation is valid for the hedging conditional.²³³
3. The hedging conditional cannot be true if the antecedent is true and the consequent is untrue.

Let ‘ \rightsquigarrow ’ represent the hypothetical hedging conditional. So, by (1) and Modes Ponens for the material conditional, for any sentences A and B,

$$((A \rightarrow B) \cdot A) \rightsquigarrow B$$

By (2) we get

$$(A \rightarrow B) \rightsquigarrow (A \rightsquigarrow B)$$

²³³ For the purposes of this argument, I only need one direction of exportation: ‘if P and Q then R’ implies ‘if P then if Q then R.’

By this and (3) it follows that the hedging conditional is as weak as the material conditional.²³⁴ I think that the obvious point of criticism will be with (2), however that response would be ad hoc without a principled limitation on exportation. Exportation is intuitively valid when hedging. Consider (a schizophrenic's admission?): 'if Carl and Dana both exist then I hear them talk to each other.' That necessitates 'if Carl exists then if Dana exists then I hear them talk to each other.'

Larry Powers contends that this argument amounts to demonstration that a conditional with exportation *just is* a material conditional.²³⁵ He points out that other (known) conditionals reject exportation, all for principled reasons. Exportation is invalid for strict conditionals, counterfactual conditionals, and relevance conditionals. If the hedging conditional could not handle half-hedging if exportation were valid, then that itself would be a principled reason for giving up exportation. This seems plausible. At least then, the above argument should be a *prima facie* reason not to search for a hedging conditional; intuitively, arguments with hedging conditionals and the form of exportation are valid, but a hedging conditional would have to invalidate them.

In addition to an abandonment of exportation, there is some intuitive pressure for a hedging conditional to give up contraposition. Consider the sentence:

If Santa exists then it is not the case that Santa wears a green suit.

By the rule contraposition, that sentence is equivalent to

If Santa wears a green suit then Santa does not exist.

There are two options: (1) reject contraposition for \Rightarrow or (2) defend the equivalence (or the top-to-bottom implication) in the same way one defends contraposition for the material conditional

²³⁴ I have seen this argument somewhere before about some other conditional but I cannot remember or find where.

²³⁵ In conversation.

against its ‘counterexamples’²³⁶—the second sentence must have either a true consequent or a false antecedent if the first is true—but this is just to treat the hedging conditional as a material conditional.

Even if these arguments are successful, they do not seem to get at the real problem with hedging in the object language. That problem is that is the same as what recommends free logic in the first place: sentence with non-referring names do not express something that could be true or false. I think the knock-down argument against hedging relies on that point, but that point is not sufficient to get hedgers to abandon their intuitions. So, I will just provide a criticism of the only well worked out account of hedging that I know: Pryor’s 2006b.

Pryor’s Account

James Pryor in his unpublished paper ‘More on Hyper-Reliability and A Priority’ (2006b) presents a semantics for a neutral free logic that is intended to deal favorably with intuitions of hedged and partially-hedged sentences. I argue below that the semantics he presents is inadequate to capture the intuitions about hedging that he wishes to preserve.

The test sentence:

(TS) If Jack exists then Jack is younger than Jiho.

Here is the intuition that guides Pryor’s proposed semantics: The test sentence is hedged against Jack’s non-existence so should imply his existence. It is not hedged against Jiho’s non-existence so it should imply Jiho’s existence. Below is the formalization of the test sentence (FTS) that Pryor settles on as being best for capturing that intuition:

(FTS) $\forall x[(Jack = x) \rightarrow (x < Jiho)]$

The crucial features of his semantic account that give the result are:

(1) It allows neutrality for some sentences with empty names.

²³⁶ For example, Frank Jackson’s ‘If it rains then it will not rain heavily’ seems to defy contraposition.

- (2) It has strong Kleene tables.
- (3) Quantifiers are trivalent.
- (4) Reference failure poisons (neutralizes) identity statements and atomic relations.
- (5) The rule for \forall takes the minimum value of all assignments.

Pryor also argues that sentences in his preferred hedging form will not imply the existence of the thing hedged against, but *can* or *may* imply the existence of things not hedged against.²³⁷ I do not share Pryor's intuitions about the existential implications of the test sentence. However my goal is to show that while his semantic account *seems* to capture his intuitions about the test sentence it is insufficient to account for similar intuitions about hedging, and in fact, it is insufficient to accommodate his own guiding intuition. There are two conceivable kinds of counterexample to a semantics designed in part to accommodate hedging and partial hedging: (a) sentences that do not imply the existence of unhedged components, and (b) sentences that do imply the existence of hedged components. Consider first a potential case of the former:

If the test sentence (TS) should entail the existence of Jiho, it is because it is not hedged against the possible non-existence of Jiho. But neither is,

If Jack exists then either he is self-identical or he is younger than Jiho.

Symbolizing it in the manner recommended by Pryor gives,

$$\star \quad \forall x[(Jack = x) \rightarrow \{(x = x) \vee (x < Jiho)\}].$$

Whether or not 'Jack' or 'Jiho' refers, \star comes out true. So it does not imply the existence of Jiho. The culprit is (2), the strong Kleene table. In fact, there are even simpler transgressions. Intuitively, since it is un-hedged the following ought to imply that Jiho exists:

Jack is short or he is taller than Jiho.

²³⁷ Pryor, "More on hyper-reliability and a priority," 12.

Obviously, it does not imply that Jiho exists if ‘or’ is symbolized as a strong Kleene disjunction. So there are un-hedged sentences that do not imply the existence of their constituents. That is sufficient, I think, to conclude that the intuitive hedging datum is not decisive in favor of Pryor’s semantics.

However, Pryor is careful not to claim that half-hedged sentences *always* imply the existence of the unhedged components. He says,

So on this interpretation, [(FTS)] *can* finally entail *Jiho exists*. As we’ll see below, on this logic, a hedged predication $\forall x(\alpha = x \rightarrow \varphi)$, with α not occurring in φ , will never entail α *exists*. It may entail β *exists* for other terms β occurring in φ .²³⁸

Even if Pryor is satisfied with the counterintuitive result that some seemingly partially-hedged sentences are fully hedged (that partial-hedging is partially-helpful), *something* in the quoted passage is incorrect; either some sentences hedged against α ’s non-existence (α -hedged claims) still entail that α exists or else α -hedged claims *cannot* entail that β exists for other terms β occurring in φ .

To see why, consider two putative cases of hedged sentences that still seem to entail Jack’s existence.²³⁹

** $\forall x[(Jack = x) \rightarrow (Jack < Jiho)]$

If Jack does not refer then by (2) and (4), all assignments of domain elements to x result in a valuation of neutral. So by (5) the displayed sentence ** would be neutral. That is, ** could not be true unless $\exists x(Jack = x)$ was true. If you share Pryor’s intuitions about the test sentence,

²³⁸ Ibid.

²³⁹ These sentences do not really entail Jack’s existence on Pryor’s semantics. I will explain fully below, but the general idea of my criticism is that to avoid these sorts of counterexamples, Pryor must resort to a treatment of the empty domain that trivializes hedging.

then intuitively, $\star\star$ should be hedged against Jack's non-existence, so it should not entail his existence. In fact, $\star\star$ is a more natural formalization of the test sentence than Pryor's symbolization; his seems to be a symbolization of:

If Jack exists then *he* is younger than Jiho.

Nevertheless, Pryor should say that $\star\star$ is a misrepresentation of the test sentence (TS), and (against intuition) that whatever English sentence it represents (if any) is not hedged against Jack's non-existence (despite its similarity to the hedged test sentence).

Here is a second potential case of an intuitively hedged sentence that should not, but still (seemingly²⁴⁰) implies the existence of Jack.

If Jack exists then he is identical to Kyle but not to Lin.

Here is the appropriately formalized version (this one *Jack*-hedged since 'Jack' does not appear in the consequent):

$$\star\star\star \quad \forall x[(Jack = x) \rightarrow (x = Kyle \cdot x \neq Lin)]$$

If there are no models of $\star\star\star$ which are not also models of $\exists x(Jack = x)$ then the latter is implied by the former and the attempted hedge fails. Consider a model \mathbf{M} without $\exists x(Jack = x)$. By (4), every assignment of domain elements to variables will make the antecedent neutral. By (2) and (5), $\star\star\star$ will be true on \mathbf{M} (and not neutral) just in case the consequent is true on every assignment of domain elements to x . If 'Kyle' does not pick out a domain element then (by (2) and (4)) the consequent will be neutral on all assignments, so suppose $\exists x(Kyle = x)$. Now, either 'Lin' is empty or not. If it is empty then by (2) and (4) the consequent will be neutral. If it is not empty, then there will be an assignment of domain elements to x , namely the referent of 'Lin,' that makes the consequent false. So, apparently, there are *Jack*-hedged

²⁴⁰ It does imply the existence of Jack on Pryor's semantics if the empty domain is ignored.

sentences (in Pryor's recommended form—without 'Jack' in the consequent) that still imply Jack's existence.

However, Pryor gives a proof that on his semantics this cannot be! He proves that no α -hedged sentence can entail that α exists. That demonstration relies on two key features of his semantic account that I have not yet mentioned:

(6) The domain is possibly empty.

(7) $\forall\varphi$ is true on a model if the domain is empty.

Here is Pryor's proof that a hedged predication $\forall x(\alpha = x \rightarrow \varphi)$, with α not occurring in φ , will never entail α exists:

Let M be any model assigning nothing to α , but where every object in its domain satisfies φ (if φ is something like $x \neq x$, this requires giving M an empty domain). Then $|\alpha = x \rightarrow \varphi|_V^M$ will be T wrt every assignment V, and so $|\forall x(\alpha = x \rightarrow \varphi)|^M$ will be T, but $|\alpha$ exists $|^M$ non-T. So $\forall x(\alpha = x \rightarrow \varphi)$ cannot entail α exists.²⁴¹

It is true that $\forall x(\alpha = x \rightarrow \varphi)$ can always be satisfied by the empty domain since $|\forall x\varphi|_V^M$ is true when M's domain is empty!²⁴² So of course it follows that an α -hedged claim cannot entail that α exists. In fact, no existentially quantified claim follows from any universally quantified claim because there is a model—one with the empty domain—that makes the former claim true and the latter false. So, on Pryor's semantics, the addition of even a *vacuous* universal quantifier hedges against the existence of *anything*.

Unfortunately, the universal quantifier is much too blunt an instrument for the hedging that Pryor wants. The appeal to the empty domain in Pryor's proof above, means that the proof

²⁴¹ Ibid., 16.

²⁴² Ibid., 13.

makes no use of the antecedent in the α -hedged sentence. The hedging effect is wholly unrelated to what intuitively does the hedging—the antecedent of the conditional. As a consequence, no *Jack*-hedged sentence (not even (FTS)) *can* entail that Jiho exists or that β exists for any β .

Some partial hedging could be saved by requiring a non-empty domain (or possibly by tinkering with the \forall rule for an empty domain so that empty names are poisonous). However, this would have the effect of rendering α -hedging unreliable; it would allow cases like *** that are α -hedged yet still entail the existence of α . Consequently, the sort of hedging that Pryor is after is not possible on the neutral free semantics that he proposes in “More on hyper-reliability and a priority;” hedging is either too heavy-handed or not successful enough.

Use and Mention

Perhaps tinkering could save the hedging intuitions. However, I think it would be best to abandon the effort and explain the intuitions away. Maybe the intuition that we ought to be able to hedge against non-existence stems from the scarcity of circumstances in which asserting a conditional implies the assertion of its antecedent. However rare, it does sometimes happen; in classical logic these cases arise when the antecedent is tautological.²⁴³ The semantics I argue for in this chapter, nicely puts (TS) and similar sentences among the conditional sentences that could not have false antecedents, so it is appropriate that (TS) entails its antecedent (though the conditional corresponding to the inference is not a logical truth).

Intuitions might also be explained by appeal to a sort of use/mention confusion. When we take (TS) in such a way that it does not imply the existence of Jack we take the occurrence of ‘Jack’ to stand in for a description like ‘the thing named ‘Jack.’ That is, we understand (TS) to mean:

²⁴³ Or in Curry sentences where the conditional sentence is its own antecedent. For example let S be the sentence: If S is true then WSU is in Siberia. The truth of the sentence implies the truth of the antecedent.

(TS') If there is a referent of 'Jack' then it is younger than Jiho.

Call (TS') the *mention* interpretation of (TS). This gets the simplest intuitions about the test sentence right, if the metalanguage has the same characteristics as an object language with the semantic scheme presented in this chapter. (TS') entails the existence of Jiho, since every model where 'Jiho' does not refer gives the conditional a neutral consequent. If 'Jiho' does refer then the sentence is vacuously true if 'Jack' does not refer and false only if 'Jack' does refer and has a referent that is not younger than Jiho.

The *mention* interpretation also avoids the problem of contingent a priori existence claims. Take for example the sentence:

If Santa exists then either he is an elf or not.

The *mention* interpretation is:

If 'Santa' has a referent then it is either an elf or not.

Letting \mathcal{S} represent the set of all values of $\mathbf{d}('Santa')$ if any, the meta-linguistic symbolization is

$$\forall x[(x \in \mathcal{S}) \rightarrow (Ex \vee \sim Ex)]$$

This sentence is seemingly a priori yet does not entail the (contingent) existence of Santa.

One might be initially inclined to represent the metalinguistic sentence as:

$$\forall x[(\mathbf{d}('Santa') = x) \rightarrow (Ex \vee \sim Ex)]$$

However, if $\mathbf{d}('Santa')$ is undefined and the metalanguage has the same general rules as the object language then the entire sentence would be neutral—so it would imply the existence of Santa after all.

Since the mention interpretation handles half-hedging there is no problem with exportation and no discomfort with contraposition. For example, on the mention interpretation,

contraposing 'If Santa exists then it is not the case that he wears a green suit' gives the unproblematic,

If anything wears a green suit then it is not the referent of 'Santa'.

The trouble cases for Pryor's semantics are similarly handled by appeal to the mention interpretation. I conclude, perhaps too hastily, that the hedging intuition rests on a use/mention confusion (similar to the confusion that underlies the intuition that 'Santa does not exist' can be true).

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ABSTRACT**AN ARGUMENT FOR A NEUTRAL FREE LOGIC**

by

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Advisor: Michael McKinsey**Major:** Philosophy**Degree:** Doctor of Philosophy

I argue for a neutral free logic, which is a logic wherein sentences containing non-referring terms do not have truth value. The primary support for this conclusion comes by way of criticism of the alternatives. If every sentence of the form ‘ $a = a$ ’ is a logical truth and is consequently knowable a priori then it will follow absurdly that ‘ a exists’ is knowable a priori. There are several alternatives for avoiding this intolerable conclusion and I argue that, with the exception of neutral free logic, which holds that ‘ $a = a$ ’ can lack truth value, their successes are not sufficient to outweigh their shortcomings.

One option is to reject the closure of a priori knowability. However, there are no plausible counterexamples to a carefully stated closure principle. Another option is to try to avoid the conclusion by rejecting the validity of ‘something is a , so a exists.’ However, this response, in its strongest form relies on an implausible ambiguity in the quantifiers ‘all’ and ‘some.’ One could avoid the intolerable conclusion if ‘ $a = a$ ’ does not imply that ‘something is a .’ There are two main possibilities for this approach: positive free logic and supervaluational logic. The former absurdly abandons one of the most obviously valid argument forms in the

history of the study of logic. The latter, despite its technical sophistication and apparent utility, mischaracterizes truth. Furthermore, I argue that the intuitions that recommend supervaluational semantics can be explained by appeal only to resources available to the neutral free logician. Also, the intolerable conclusion might be avoided by maintaining that 'a = a' is false rather than truth-valueless (or neutral). Such a logical system is a negative free logic. Its primary support comes from the principle of bivalence. I argue that bivalence and its syntactic relative, the law of excluded middle, are not well justified. The only remaining alternative for avoiding the intolerable conclusion is neutral free logic. There are several possible varieties of neutral free logic that, very roughly stated, vary with respect to how permissive they are of true statements containing non-referring names. I argue for the least possible permissivity, and offer criticism of the intuitions that suggest the more permissive stances.

AUTOBIOGRAPHICAL STATEMENT

Daniel Yeakel was born to Warren and Laura Yeakel in Chattanooga, Tennessee in 1977. After spending two years in Saudi Arabia, he grew up with sister Sarah in south-eastern Michigan, graduated with a BA in philosophy from the University of Michigan—Dearborn in 2001 and with an MA in philosophy from Wayne State University in 2005. He is married to a lovely wife, Jessica and is the proud father of two often-wonderful children: James and Kathryn.